Term Structure of Interest Rates with Short-Run and Long-Run Risks

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Motivation

- The failure of the expectation hypothesis is well documented: Fama and Bliss (1987), Campbell and Shiller (1991),...
- Uncovering the underlying sources of predictability is important and challenging
- Various factors are prosed to capture predictability: forward spread, CP factor, jump risk factor, hidden factor, macro-variables
- Question remains what particular economic mechanism is behind bond return predictability
- We propose a LRR-type model promising to to capture time variation of bond returns

What we do: Big Picture

- Build on Bansal and Yaron (2004) and Bollersellev, Tauchen, and Zhou (2009) and take them to bond markets
- Long-run risk model where volatility of the endowment growth and volatility of volatility (vol-of-vol) are time-varying
- Bond risk premium is time-varying: capture two sources of volatility: short-run (vol-of-vol) and long-run (vol of growth)
- Bansal and Shaliastovich (2013): LRR model helps explaining long-horizon bond premia, but no vol-of-vol factor, which helps pinning down the variation in bond returns due to changes in uncertainty
- We provide a structural framework for the support of the two-factor volatility models already appeared in the literature, e.g., Adrian and Rosenberg (2008), Christoffersen, Jacobs, Ornthanalai, Wang (2008), Branger, Rodrigues, Schlag (2011), and Zhou and Zhu (2012, 2013)

Empirical Factors

- The vol-of-vol factor is proxied by the variance risk premium
- The consumption growth volatility factor is proxied by forward rate-based predictors

Key results

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    - Appear to have been explained by the VRP derived from the interest rate swaptions market
    - Thus vol-of-vol (uncertainty) factor appears to drive the variation in short-horizon Treasury excess returns
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- Long-horizon (1-year) Treasury excess returns:
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- Model calibration: upward-sloping yield curve

Preferences and Economy Dynamics

Epstein-Zin recursive preferences with early resolution of uncertainty:

$$U_t = \left[(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(\mathbb{E}_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

- Early resolution of uncertainty: $\psi > 1$, $\psi > 1/\gamma$, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}} < 0$
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$$\begin{aligned} x_{t+1} &= \rho_x x_t + \phi_e \sigma_{g,t} z_{x,t+1}, \\ g_{t+1} &= \mu_g + x_t + \sigma_{g,t} z_{g,t+1}, \\ \sigma^2_{g,t+1} &= a_\sigma + \rho_\sigma \sigma^2_{g,t} + \sqrt{q_t} z_{\sigma,t+1}, \\ q_{t+1} &= a_q + \rho_q q_t + \phi_q \sqrt{q_t} z_{q,t+1}. \end{aligned} \qquad (2)$$

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$$\begin{aligned} & m_{t+1} - \mathbb{E}_t[m_{t+1}] = \\ & -\lambda_g \sigma_{g,t} z_{g,t+1} - \lambda_x \sigma_{g,t} z_{x,t+1} - \lambda_\sigma \sqrt{q_t} z_{\sigma,t+1} - \lambda_q \sqrt{q_t} z_{q,t+1} \end{aligned} \tag{3}$$

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- The market prices of risk: λ_g, λ_x, λ_σ, λ_q

$$\begin{array}{ll} \lambda_g = \gamma > 0 & \lambda_\sigma = (\theta-1)\kappa_1 A_\sigma \phi_\sigma < 0 \\ \lambda_x = (\theta-1)\kappa_1 A_x \phi_e > 0 & \lambda_q = (\theta-1)\kappa_1 A_q \phi_q < 0 \end{array} \tag{4}$$

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Inflation

We conjecture inflation process:

$$\pi_{t+1} = a_\pi + \rho_\pi \pi_t + \phi_\pi z_{\pi,t+1} + \phi_{\pi g} \sigma_{g,t} z_{g,t+1} + \phi_{\pi\sigma} \sqrt{q_t} z_{\sigma,t+1} \tag{5}$$

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- Endowment growth shock $z_{g,t+1}$ shock (real side)
- Endowment growth vol $z_{\sigma,t+1}$ shock (real side)
- Thus, in our model real side shocks affect inflation: money non-neutrality, support by Pennacchi (1991), Sun (1992), Zhou (2011)

Bond pricing

A general recursion for $P_t^{\$,n}$: $n-$period zero-coupon bond at time t:

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The (log) price of the $n-$period nominal bond $p_t^{\$,n}$ is affine in state variables x_t, σ_t^2, q_t, and inflation π_t:

$$p_t^{\$,n} = B_0^{\$,n} + B_1^{\$,n} x_t + B_2^{\$,n} \sigma_t^2 + B_3^{\$,n} q_t + B_4^{\$,n} \pi_t \tag{7}$$

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Solve for $B_0^{\$,n}$, $B_1^{\$,n}$, $B_2^{\$,n}$, $B_3^{\$,n}$, $B_4^{\$,n}$ recursively

And obtain closed form solutions for bond prices

Nominal risk-free rate

$$
\begin{aligned}
r^{\$}_{f,t} = {} & -\theta \ln \delta + \gamma \mu_g + a_\pi - (\theta - 1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] - \frac{1}{2}\phi_\pi^2 \\
& + [\gamma - (\theta - 1)A_x(\kappa_1 \rho_x - 1)]\, x_t \\
& + \left[-(\theta - 1)A_\sigma(\kappa_1 \rho_\sigma - 1) - \frac{1}{2}\gamma^2 - \frac{1}{2}(\theta - 1)^2(\kappa_1 A_x \phi_e)^2 - \frac{1}{2}\phi_{\pi g}^2 - \gamma \phi_{\pi g}\right] \sigma^2_{g,t} \\
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&+ \left[((\theta - 1)\kappa_1 A_\sigma - \phi_{\pi\sigma})(B_2^{\$,n-1} + B_4^{\$,n-1}\phi_{\pi\sigma}) * +(\theta - 1)\kappa_1 A_q B_3^{\$,n-1}\phi_q^2\right] q_t \\
&- B_4^{\$,n-1}\phi_\pi^2 \\
&\equiv \beta_1^{\$,n-1}\sigma_{g,t}^2 + \beta_2^{\$,n-1} q_t - B_4^{\$,n-1}\phi_\pi^2
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\text{brp}_{t+1}^{\$,n-1} &= \text{Cov}_t[m_{t+1}^{\$}, p_{t+1}^{\$,n-1}] \\
&= \left[-(\gamma + \phi_{\pi g})B_4^{\$,n-1}\phi_{\pi g} + (\theta - 1)\kappa_1 A_x B_1^{\$,n-1}\phi_e^2\right] \sigma_{g,t}^2 \\
&+ \left[((\theta - 1)\kappa_1 A_\sigma - \phi_{\pi\sigma})(B_2^{\$,n-1} + B_4^{\$,n-1}\phi_{\pi\sigma}) * +(\theta - 1)\kappa_1 A_q B_3^{\$,n-1}\phi_q^2\right] q_t \\
&- B_4^{\$,n-1}\phi_\pi^2 \\
&\equiv \beta_1^{\$,n-1}\sigma_{g,t}^2 + \beta_2^{\$,n-1} q_t - B_4^{\$,n-1}\phi_\pi^2
\end{aligned} \tag{9}$$

- Stochastic vol premium is amplified by $\phi_{\pi g}$
- Vol-of-vol premiums is amplified by $\phi_{\pi\sigma}$
- Autonomous inflation shock is captured by the last term

Variance risk premium in the model

$$\begin{aligned} \mathrm{VRP}_t &= \mathbb{E}_t^{\mathbb{Q}}\left[\sigma_{r,t+1}^2\right] - \mathbb{E}_t^{\mathbb{P}}\left[\sigma_{r,t+1}^2\right] \\ &= (\theta - 1)\kappa_1 \left[A_\sigma(1 + \kappa_1^2 A_x^2 \phi_e^2) + A_q \kappa_1^2 \phi_q^2 (A_\sigma^2 + A_q^2 \phi_q^2)\right] q_t \end{aligned} \tag{10}$$

Variance risk premium in the model

$$\begin{aligned}\text{VRP}_t &= \mathbb{E}_t^{\mathbb{Q}}\left[\sigma^2_{r,t+1}\right] - \mathbb{E}_t^{\mathbb{P}}\left[\sigma^2_{r,t+1}\right] \\ &= (\theta-1)\kappa_1\left[A_\sigma(1+\kappa_1^2A_x^2\phi_e^2)+A_q\kappa_1^2\phi_q^2(A_\sigma^2+A_q^2\phi_q^2)\right] q_t\end{aligned} \tag{10}$$

- Vol-of-vol q_t factor is the only factor that drives variance risk premium

Variance risk premium in the model

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- Vol-of-vol q_t factor is the only factor that drives variance risk premium
- $\theta < 0$, $\kappa_1 > 0$ and $A_\sigma < 0$, $A_q < 0$ ensure positive loading of VRP on q_t

Variance risk premium in the model

$$\begin{aligned} \mathrm{VRP}_t &= \mathbb{E}_t^{\mathbb{Q}}\left[\sigma_{r,t+1}^2\right] - \mathbb{E}_t^{\mathbb{P}}\left[\sigma_{r,t+1}^2\right] \\ &= (\theta-1)\kappa_1\left[A_\sigma(1+\kappa_1^2 A_x^2\phi_e^2) + A_q\kappa_1^2\phi_q^2(A_\sigma^2 + A_q^2\phi_q^2)\right] q_t \end{aligned} \tag{10}$$

- Vol-of-vol q_t factor is the only factor that drives variance risk premium
- $\theta < 0$, $\kappa_1 > 0$ and $A_\sigma < 0$, $A_q < 0$ ensure positive loading of VRP on q_t
- Construct VRP from interest swaptions data and
- Assess what fraction of Treasury excess return variation can be attributed to the vol-of-vol factor (proxied by VRP) and what fraction is attributable to the forward spread factor

Variance risk premium: construction

$\mathbb{E}_t^{\mathbb{Q}}\left[\sigma_{r,t+1}^2\right]$:

- $\mathbb{Q}$-expectation of the variance (risk-neutral, implied), methodology by Li and Song (2013)
- Data: 1-month options on 10-year interest rate swap rates
- Source: Barclays Capital

Details on swaptions

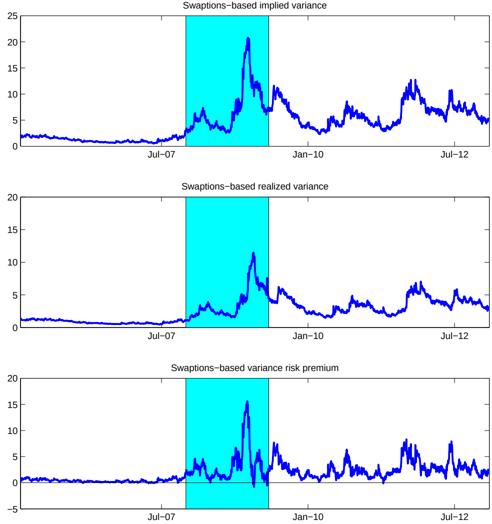
$\mathbb{E}_t^{\mathbb{P}}\left[\sigma_{r,t+1}^2\right]$:

- $\mathbb{P}$-expectation of the variance (physical, realized)
- Data: 10-year swap rates' 5-min quadratic variations aggregated into daily quadratic variation
- Source: Bloomberg

Details on $\mathbb{P}$-measure construction

Sample period: 2005-2012, daily frequency

Swaptions-implied Variance Risk Premium



Baseline regression

To assess the predictability content of the variance risk premium, we run the following regressions:

$$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)VRP_t + \beta_2^{(\tau)}(h)FS_t^{(\tau)} + CP_t + \epsilon_{t+h}^{(\tau)}, \tag{11}$$

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LHS: Treasury bond excess returns:

$$rx_{t+h}^{(\tau)} = r_{t+h}^{(\tau)} - y_t^{(h)} = p_{t+h}^{(\tau-h)} - p_t^{(\tau)} - y_t^{(h)} \tag{12}$$

$p_t^{(\tau)}$ is the time-t log bond price of a τ-period bond;
$y_t^{(h)}$ is the time-t $h-$period bond yield

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To assess the predictability content of the variance risk premium, we run the following regressions:

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$p_t^{(\tau)}$ is the time-t log bond price of a τ-period bond;
$y_t^{(h)}$ is the time-t $h-$period bond yield

RHS: Explanatory variables:

- Variance risk premium, VRP_t
- Fama-Bliss forward spread, $FS_t^{(\tau)} = f_t^{(\tau-h\rightarrow\tau)} - y_t^{(h)}$
- Cochrane-Piazzesi factor CP_t, forward rates factor

Table: 1-month Treasury excess returns, 2yr maturity

Const	-0.023	-0.019	-0.028	-0.026	-0.031	-0.031
	(-4.01)	(-6.18)	(-3.66)	(-8.53)	(-4.44)	(-9.09)
VRP	0.005			0.003	0.004	0.003
	(3.00)			(3.98)	(2.52)	(3.10)
FS		2.762		2.405		2.212
		(6.10)		(7.18)		(7.15)
CP			0.670		0.480	0.311
			(3.14)		(2.61)	(2.87)
Adj. R2	28.86	53.88	27.73	67.71	40.82	72.51

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	(-4.01)	(-6.18)	(-3.66)	(-8.53)	(-4.44)	(-9.09)
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	(3.00)			(3.98)	(2.52)	(3.10)
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Const	-0.023 (-4.01)	-0.019 (-6.18)	-0.028 (-3.66)	-0.026 (-8.53)	-0.031 (-4.44)	-0.031 (-9.09)
VRP	0.005 (3.00)			0.003 (3.98)	0.004 (2.52)	0.003 (3.10)
FS		2.762 (6.10)		2.405 (7.18)		2.212 (7.15)
CP			0.670 (3.14)		0.480 (2.61)	0.311 (2.87)
Adj. R2	28.86	53.88	27.73	67.71	40.82	72.51

Table: 1-month Treasury excess returns, 3yr maturity

Const	-0.022	-0.025	-0.027	-0.030	-0.031	-0.034
	(-3.70)	(-6.38)	(-3.61)	(-8.67)	(-4.31)	(-9.31)
VRP	0.005			0.003	0.003	0.003
	(2.91)			(2.86)	(2.39)	(2.38)
FS		1.762		1.547		1.412
		(6.51)		(7.65)		(7.71)
CP			0.690		0.507	0.267
			(3.25)		(2.76)	(2.67)
Adj. R2	25.38	55.69	26.72	65.46	37.52	68.37

Table: 1-month Treasury excess returns, 3yr maturity

Const	-0.022 (-3.70)	-0.025 (-6.38)	-0.027 (-3.61)	-0.030 (-8.67)	-0.031 (-4.31)	-0.034 (-9.31)
VRP	0.005 (2.91)			0.003 (2.86)	0.003 (2.39)	0.003 (2.38)
FS		1.762 (6.51)		1.547 (7.65)		1.412 (7.71)
CP			0.690 (3.25)		0.507 (2.76)	0.267 (2.67)
Adj. R2	25.38	55.69	26.72	65.46	37.52	68.37

Table: 1-month Treasury excess returns, 3yr maturity

Const	-0.022	-0.025	-0.027	-0.030	-0.031	-0.034
	(-3.70)	(-6.38)	(-3.61)	(-8.67)	(-4.31)	(-9.31)
VRP	0.005			0.003	0.003	0.003
	(2.91)			(2.86)	(2.39)	(2.38)
FS		1.762		1.547		1.412
		(6.51)		(7.65)		(7.71)
CP			0.690		0.507	0.267
			(3.25)		(2.76)	(2.67)
Adj. R2	25.38	55.69	26.72	65.46	37.52	68.37

Table: 1-month Treasury excess returns, 4yr mat

Const	-0.021 (-3.36)	-0.028 (-6.73)	-0.027 (-3.52)	-0.034 (-10.54)	-0.030 (-4.12)	-0.038 (-11.95)
VRP	0.005 (2.68)			0.003 (4.37)	0.003 (2.15)	0.003 (3.33)
FS		1.309 (6.85)		1.184 (8.13)		1.093 (8.03)
CP			0.705 (3.30)		0.526 (2.82)	0.270 (2.77)
Adj. R2	21.04	51.53	23.42	61.87	31.94	64.30

Table: 1-month Treasury excess returns, 4yr mat

Const	-0.021	-0.028	-0.027	-0.034	-0.030	-0.038
	(-3.36)	(-6.73)	(-3.52)	(-10.54)	(-4.12)	(-11.95)
VRP	0.005			0.003	0.003	0.003
	(2.68)			(4.37)	(2.15)	(3.33)
FS		1.309		1.184		1.093
		(6.85)		(8.13)		(8.03)
CP			0.705		0.526	0.270
			(3.30)		(2.82)	(2.77)
Adj. R2	21.04	51.53	23.42	61.87	31.94	64.30

Table: 1-month Treasury excess returns, 4yr mat

Const	-0.021 (-3.36)	-0.028 (-6.73)	-0.027 (-3.52)	-0.034 (-10.54)	-0.030 (-4.12)	-0.038 (-11.95)
VRP	0.005 (2.68)			0.003 (4.37)	0.003 (2.15)	0.003 (3.33)
FS		1.309 (6.85)		1.184 (8.13)		1.093 (8.03)
CP			0.705 (3.30)		0.526 (2.82)	0.270 (2.77)
Adj. R2	21.04	51.53	23.42	61.87	31.94	64.30

Table: 1-month Treasury excess returns, 5yr mat

	(-3.33)	(-8.31)	(-3.46)	(-9.90)	(-4.13)	(-10.36)
VRP	0.005			0.002	0.004	0.002
	(3.04)			(1.74)	(2.51)	(1.44)
FS		1.168		1.051		0.977
		(8.06)		(7.32)		(6.98)
CP			0.738		0.542	0.232
			(3.38)		(2.81)	(2.04)
Adj. R2	20.58	57.09	21.66	59.46	30.29	60.7

Table: 1-month Treasury excess returns, 5yr mat

	(-3.33)	(-8.31)	(-3.46)	(-9.90)	(-4.13)	(-10.36)
VRP	0.005 (3.04)			0.002 (1.74)	0.004 (2.51)	0.002 (1.44)
FS		1.168 (8.06)		1.051 (7.32)		0.977 (6.98)
CP			0.738 (3.38)		0.542 (2.81)	0.232 (2.04)
Adj. R2	20.58	57.09	21.66	59.46	30.29	60.7

Table: 1-month Treasury excess returns, 5yr mat

	(-3.33)	(-8.31)	(-3.46)	(-9.90)	(-4.13)	(-10.36)
VRP	0.005			0.002	0.004	0.002
	(3.04)			(1.74)	(2.51)	(1.44)
FS		1.168		1.051		0.977
		(8.06)		(7.32)		(6.98)
CP			0.738		0.542	0.232
			(3.38)		(2.81)	(2.04)
Adj. R2	20.58	57.09	21.66	59.46	30.29	60.7

Table: 3-month Treasury excess returns, 2yr mat

Const	-0.015 (-2.95)	-0.013 (-4.48)	-0.022 (-3.11)	-0.018 (-5.41)	-0.024 (-3.54)	-0.023 (-6.64)
VRP	0.004 (2.54)			0.002 (3.34)	0.002 (1.99)	0.002 (2.43)
FS		2.353 (5.40)		2.113 (5.93)		1.925 (6.08)
CP			0.590 (2.92)		0.463 (2.57)	0.318 (3.10)
Adj. R2	19.16	49.98	26.81	57.39	32.91	63.59

Table: 3-month Treasury excess returns, 2yr mat

Const	-0.015	-0.013	-0.022	-0.018	-0.024	-0.023
	(-2.95)	(-4.48)	(-3.11)	(-5.41)	(-3.54)	(-6.64)
VRP	0.004			0.002	0.002	0.002
	(2.54)			(3.34)	(1.99)	(2.43)
FS		2.353		2.113		1.925
		(5.40)		(5.93)		(6.08)
CP			0.590		0.463	0.318
			(2.92)		(2.57)	(3.10)
Adj. R2	19.16	49.98	26.81	57.39	32.91	63.59

Table: 3-month Treasury excess returns, 2yr mat

Const	-0.015	-0.013	-0.022	-0.018	-0.024	-0.023
	(-2.95)	(-4.48)	(-3.11)	(-5.41)	(-3.54)	(-6.64)
VRP	0.004			0.002	0.002	0.002
	(2.54)			(3.34)	(1.99)	(2.43)
FS		2.353		2.113		1.925
		(5.40)		(5.93)		(6.08)
CP			0.590		0.463	0.318
			(2.92)		(2.57)	(3.10)
Adj. R2	19.16	49.98	26.81	57.39	32.91	63.59

Table: 3-month Treasury excess returns, 3yr mat

Const	-0.012 (-1.95)	-0.018 (-4.20)	-0.019 (-2.50)	-0.020 (-4.48)	-0.021 (-2.74)	-0.024 (-5.49)
VRP	0.003 (2.16)			0.001 (1.82)	0.002 (1.52)	0.001 (1.14)
FS		1.633 (5.56)		1.530 (5.81)		1.402 (5.96)
CP			0.600 (2.70)		0.500 (2.42)	0.269 (2.60)
Adj. R2	10.24	46.73	19.11	48.28	21.16	50.87

Table: 3-month Treasury excess returns, 3yr mat

Const	-0.012	-0.018	-0.019	-0.020	-0.021	-0.024
	(-1.95)	(-4.20)	(-2.50)	(-4.48)	(-2.74)	(-5.49)
VRP	0.003			0.001	0.002	0.001
	(2.16)			(1.82)	(1.52)	(1.14)
FS		1.633		1.530		1.402
		(5.56)		(5.81)		(5.96)
CP			0.600		0.500	0.269
			(2.70)		(2.42)	(2.60)
Adj. R2	10.24	46.73	19.11	48.28	21.16	50.87

Table: 3-month Treasury excess returns, 3yr mat

Const	-0.012	-0.018	-0.019	-0.020	-0.021	-0.024
	(-1.95)	(-4.20)	(-2.50)	(-4.48)	(-2.74)	(-5.49)
VRP	0.003			0.001	0.002	0.001
	(2.16)			(1.82)	(1.52)	(1.14)
FS		1.633		1.530		1.402
		(5.56)		(5.81)		(5.96)
CP			0.600		0.500	0.269
			(2.70)		(2.42)	(2.60)
Adj. R2	10.24	46.73	19.11	48.28	21.16	50.87

Table: 3-month Treasury excess returns, 4yr mat

Const	-0.008 (-1.06)	-0.020 (-4.21)	-0.016 (-1.92)	-0.022 (-4.21)	-0.017 (-1.99)	-0.025 (-5.13)
VRP	0.003 (1.47)			0.001 (1.53)	0.001 (0.77)	0.001 (0.67)
FS		1.318 (5.63)		1.275 (5.81)		1.192 (5.88)
CP			0.597 (2.33)		0.532 (2.15)	0.261 (1.92)
Adj. R2	3.90	39.05	11.85	39.17	11.55	40.38

Table: 3-month Treasury excess returns, 4yr mat

Const	-0.008 (-1.06)	-0.020 (-4.21)	-0.016 (-1.92)	-0.022 (-4.21)	-0.017 (-1.99)	-0.025 (-5.13)
VRP	0.003 (1.47)			0.001 (1.53)	0.001 (0.77)	0.001 (0.67)
FS		1.318 (5.63)		1.275 (5.81)		1.192 (5.88)
CP			0.597 (2.33)		0.532 (2.15)	0.261 (1.92)
Adj. R2	3.90	39.05	11.85	39.17	11.55	40.38

Table: 3-month Treasury excess returns, 4yr mat

Const	-0.008	-0.020	-0.016	-0.022	-0.017	-0.025
	(-1.06)	(-4.21)	(-1.92)	(-4.21)	(-1.99)	(-5.13)
VRP	0.003			0.001	0.001	0.001
	(1.47)			(1.53)	(0.77)	(0.67)
FS		1.318		1.275		1.192
		(5.63)		(5.81)		(5.88)
CP			0.597		0.532	0.261
			(2.33)		(2.15)	(1.92)
Adj. R2	3.90	39.05	11.85	39.17	11.55	40.38

Table: 3-month Treasury excess returns, 5yr mat

	(-0.82)	(-3.45)	(-1.49)	(-3.30)	(-1.65)	(-3.78)
VRP	0.003			-0.000	0.002	-0.000
	(1.86)			(-0.13)	(1.24)	(-0.48)
FS		1.130		1.138		1.083
		(5.02)		(4.81)		(4.78)
CP			0.617		0.514	0.182
			(2.09)		(1.77)	(1.00)
Adj. R2	4.41	33.87	8.84	33.02	9.18	32.83

Table: 3-month Treasury excess returns, 5yr mat

	(-0.82)	(-3.45)	(-1.49)	(-3.30)	(-1.65)	(-3.78)
VRP	0.003			-0.000	0.002	-0.000
	(1.86)			(-0.13)	(1.24)	(-0.48)
FS		1.130		1.138		1.083
		(5.02)		(4.81)		(4.78)
CP			0.617		0.514	0.182
			(2.09)		(1.77)	(1.00)
Adj. R2	4.41	33.87	8.84	33.02	9.18	32.83

Table: 3-month Treasury excess returns, 5yr mat

	(-0.82)	(-3.45)	(-1.49)	(-3.30)	(-1.65)	(-3.78)
VRP	0.003 (1.86)			-0.000 (-0.13)	0.002 (1.24)	-0.000 (-0.48)
FS		1.130 (5.02)		1.138 (4.81)		1.083 (4.78)
CP			0.617 (2.09)		0.514 (1.77)	0.182 (1.00)
Adj. R2	4.41	33.87	8.84	33.02	9.18	32.83

Table: 1-year Treasury excess returns, 2yr mat

Const	0.012	0.011	0.010	0.012	0.011	0.011
	(4.68)	(4.69)	(3.16)	(4.32)	(3.19)	(3.21)
VRP	-0.001			-0.001	-0.001	-0.001
	(-1.41)			(-1.66)	(-2.00)	(-2.18)
FS		-0.123		-0.035		-0.079
		(-0.39)		(-0.12)		(-0.29)
CP			0.009		0.067	0.074
			(0.09)		(0.72)	(0.81)
Adj. R2	3.38	-0.79	-1.42	2.01	3.29	2.09

Table: 1-year Treasury excess returns, 3yr mat

Const	0.027	0.024	0.023	0.026	0.025	0.025
	(6.12)	(4.41)	(4.00)	(4.47)	(4.04)	(3.75)
VRP	-0.002			-0.002	-0.002	-0.002
	(-1.68)			(-2.36)	(-2.45)	(-2.79)
FS		-0.050		0.101		0.039
		(-0.14)		(0.32)		(0.13)
CP			0.008		0.128	0.121
			(0.05)		(0.78)	(0.79)
Adj. R2	5.41	-1.35	-1.44	4.38	5.61	4.25

Table: 1-year Treasury excess returns, 4yr mat

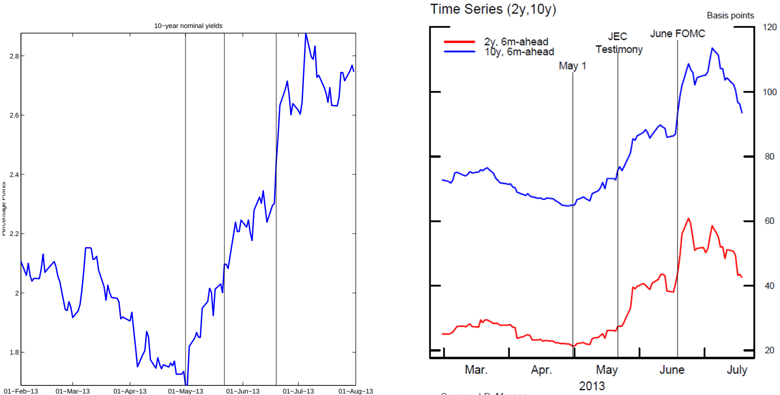
Const	0.042 (6.90)	0.032 (3.68)	0.035 (4.23)	0.037 (3.99)	0.038 (4.30)	0.035 (3.55)
VRP	-0.003 (-1.89)			-0.003 (-3.71)	-0.003 (-3.10)	-0.004 (-4.15)
FS		0.329 (0.94)		0.483 (1.55)		0.442 (1.50)
CP			0.037 (0.13)		0.237 (0.89)	0.123 (0.56)
Adj. R2	7.21	2.59	-1.38	14.13	8.45	13.49

Table: 1-year Treasury excess returns, 5yr mat

	(6.98)	(4.21)	(4.30)	(4.40)	(4.28)	(3.78)
VRP	-0.002 (-1.17)			-0.004 (-3.76)	-0.003 (-2.40)	-0.004 (-4.18)
FS		0.305 (0.89)		0.568 (1.76)		0.499 (1.78)
CP			0.196 (0.51)		0.383 (1.06)	0.212 (0.72)
Adj. R2	1.86	2.20	-0.12	10.40	4.74	10.26

Summer 2013: Taper Tantrum

- **May 1, 2013**: lowest yields in Spring-Summer 2013
- **May 22, 2013**: Fed Chairman Ben Bernanke testifies before Joint Economic Committee (Q&A): *...If we see continued improvement and we have confidence that that's going to be sustained then we could in the next few meetings ... take a step down in our pace of purchases*
- **May 22, 2013**: *the release of the May 2013 FOMC minutes: .. A number of participants expressed willingness to adjust the flow of purchases downward as early as the June meeting if the economic information received by that time showed evidence of sufficiently strong and sustained growth*
- **June 19, 2013**: *June FOMC statement and Chairman's press conference: ... the Committee may reduce the pace of purchases later this year and end the purchases in the middle of next year if economic conditions evolve as expected.*



Source: J.P. Morgan.

(a) 10-year nominal yields

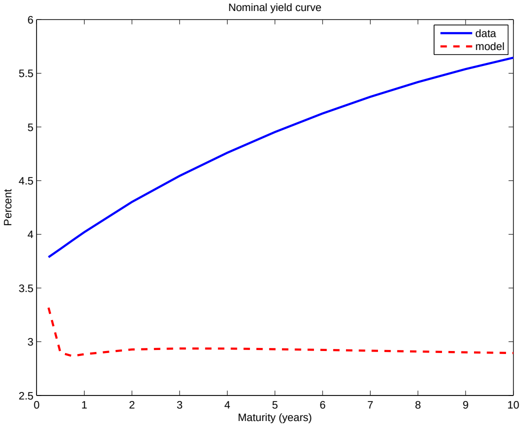
(b) Swaption implied vols

Figure: Recent Evidence: Taper Tantrum

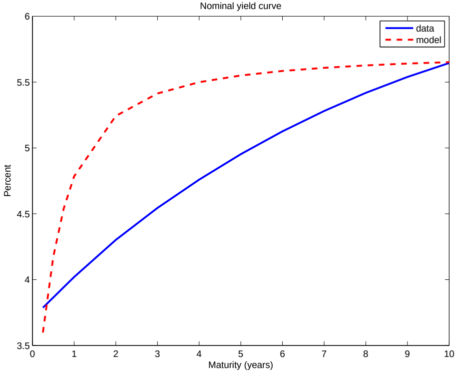
Model Calibration

Type	Param	BY	BTZ	Our choice
Panel A: Real Economy				
Preferences	δ	0.997	0.997	0.997
	γ	10	10	10
	ψ	1.5	1.5	1.5
Endowment	μ_g	0.0015	0.0015	0.0015
	ρ_x	0.979	0	0.979
	ϕ_e	0.044	0	0.044
	a_σ	0.134×10^{-5}	0.134×10^{-5}	0.134×10^{-5}
	ρ_σ	0.978	0.978	0.978
Uncertainty	a_q	n/a	2×10^{-7}	2×10^{-7}
	ρ_q	n/a	0.8	0.8
	ϕ_q	n/a	0.001	0.001
Panel B: Inflation dynamics				
Constant	a_π	n/a	n/a	12×10^{-4}
Persistence	ρ_π	n/a	n/a	0.60
Autonomous	ϕ_π	n/a	n/a	0.002
Consumption	$\phi_{\pi g}$	n/a	n/a	-0.30
Uncertainty	$\phi_{\pi\sigma}$	n/a	n/a	2.35

Term Structure in our model: no LRR risk



Term Structure in our model: with LRR risk



Conclusion

In a model with long-run risks, stochastic vol and vol-of-vol factors we show that:

- Short-horizon Treasury excess returns appear to be driven by the vol-of-vol factor proxied by the variance risk premium derived from interest rate swaptions
- Long-horizon Treasury excess returns appear to be driven more by forward spread predictors
- Both stochastic vol and vol-of-vol risks appear to be tightly linked to variation in fundamentals in the economy
- We are working on disentangling these two types of risks more explicitly in the model

- Swaptions give their holder the right, but not the obligation, to enter into an interest rate swap contract either as a fixed leg (payer swaption) or floating leg (receiver swaption) with a prespecified coupon rate K.
- The underlying of a swaption is a forward swap contract
- For date t
    - Option horizon: $T_m - t$ (option expiry T_m)
    - Tenor: $T_n - T_m$ (forward swap maturity T_n)
- So, at time t we refer to a swaption as a $(T_m - t)$ by $(T_n - T_m)$ payer swaption
- At T_m the value of a swap is par so
    - Payer swaption is equiv. to a put option with a strike of \$1 on a bond with coupon K and remaining maturity $T_m - T_n$
    - Receiver swaption is equiv. to a call swaption on the same bond

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Computation of $\mathbb{E}_t^{\mathbb{P}}(RV_{t+\tau,\tau})$:

- **Step 1:** Compute daily realized variance RV_t:

$$RV_t = \sum_{i=1}^{M} r_{t,i}^2,$$

  where $r_{t,i}$ is the 5-min log return

  - Monthly realized var $RV_{t,mon} = \frac{1}{22}\sum_{j=0}^{21} RV_{t-j}$;
  - Weekly realized var $RV_{t,week} = \frac{1}{5}\sum_{j=0}^{4} RV_{t-j}$.

- **Step 2:** Compute smoothed monthly return variance as a fitted value of:

$$RV_{t+22,mon} = \alpha + \beta_D RV_t + \beta_W RV_{t,week} + \beta_M RV_{t,mon} + \epsilon_{t+22,mon}.$$

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