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Demand coordination in a regionally integrated area

With an application to the Western Balkans

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Claude Berthomieu, Massimo Cingolani, Anastasia Ri †

Abstract

This contribution illustrates the logic of demand coordination in a regionally integrated area using cross-country demand multipliers between three separate geographical regions.

It completes an argument elegantly put forward by Muet (2004) for the special case of two countries that have the same import elasticity to income.

The argument is built by comparing different models. Initially it is assumed that the three areas have their autonomous demand fixed independently without coordination, like it is the case for "independent countries" (contrary to the case of "regions" in an integrated area). This is the assumption retained by the EU Member States when they present their national reform and their convergence programs to the European Union, which entails that spillover or feedback effects between their economic policies are neglected. The independence assumption is then relaxed by taking two of the three areas as integrated (thus making "a country"), while the third area is kept as a separate country (the "Rest of the World").

The models are kept deliberately simple to illustrate as clearly as possible the argument for fiscal policy coordination; in particular: they are linear and they exclude intermediate goods and factor movements. Although they rely on the canonical "Hansen" demand block of macroeconomic Keynesian models, which does not explicitly deal with relative prices, they can also be used to illustrate the logic of the trade-off between demand coordination and competitiveness policies.

Key words: demand coordination, saving and capital investment, macro-regional analysis, Western Balkans

- JEL codes: E61, O16, R53

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Introduction

This paper illustrates the logic of demand coordination in a regionally integrated area using cross-country demand multipliers between three separate geographical regions. It completes an argument for fiscal policy coordination put forward in an elegant example developed by Muet (2004, pp. 128-132) for the case of two countries having the same import elasticity to income. In this special case[1], the fiscal policy (government expenditures) multiplier with economic policy coordination coincides with the closed economy multiplier and is of course substantially higher than the open economy multiplier without policy coordination.

Based on a discussion of a three countries example, this contribution illustrates the result that in more general cases, the fiscal policy multiplier with coordination is less than the closed economy multiplier but higher than the multiplier without policy coordination. In other words the difference between the multiplier with and without policy coordination is positive, this difference being higher the higher the share of intra-regional trade between the areas considered, a result which was in fact recognized in rather general cases already in the 1950s (see notably Metzler, 1950), but that seems to have been meanwhile forgotten.

The argument is built by comparing models of independent economies with economies having increasing levels of trade integration. It is based on the "export" or "foreign trade multiplier", whose discovery is usually attributed to Harrod (1933) but that was discovered independently by Kalecki (1935, see King, 1998). The relevance of Harrod's foreign trade multiplier as a growth factor was confirmed empirically by the literature testing Thirlwall's laws, which are a dynamic variant of the static Harrod foreign trade multiplier.

Initially it is assumed that the three areas have their autonomous demand fixed independently without coordination, like it is the case for "independent countries" (contrary to the case of "regions" in an integrated area). This is for instance the assumption retained by the EU Member States when they present their economic reform and convergence programs to the European Union, which entails that spillover or feedback effects between their economic policies are neglected. The assumption is then relaxed by taking two of the three areas considered as integrated (in a single "country"), while the third area is taken as a separate country (the "Rest of the World").

The models are kept deliberately simple to illustrate as clearly as possible the argument for fiscal policy coordination; in particular: they are linear and there are no intermediate goods, nor factor movements. Although they rely on the canonical "Hansen" demand block of macroeconomic Keynesian models, which does not explicitly deal with relative prices, they can be used to illustrate the logic of the trade-off between demand and competitiveness policies.

The multiplier measures the effect of a change of an autonomous factor on an endogenous variable. The key variables that need to be distinguished in order to define it on the demand side are: autonomous

[1] The authors are grateful to Nadia Garbellini and Ariel Wirkierman for pointing out the implications of Muet's assumption.

demand, domestic and external, and endogenous demand, made of private consumption and export, as well as import, demand. By showing that the multiplier is higher with the coordination of autonomous policy shocks than without, the paper illustrates the opportunities that exist for gaining control over the business cycle by a positive coordination of fiscal policies, which imply also the possibility for small open economies to implement non-mercantilist full employment policies. Indeed, by increasing initially net public expenditures to the level required to bring back domestic confidence, these policies would increase also private investment and thus generate the growth that would ultimately render them sustainable from the fiscal point of view (autonomous domestic demand includes both net public expenditure and investment).

1. Models 1 to 4: three independent small open economies

Initially it is assumed that there are three regions considered to be countries, which use the same currency or whose exchange rates are fixed or stable. Their exports are exogenous, representing for each country a part of autonomous external demand.

Taking the model for country 1 :

Model 1

$$\begin{array}{llcl} \textit{Income} & Y_1 & = & Aut_1 + C_1 + X_1tR_2 + X_1tR_3 - M_1fR_2 - M_1fR_3 \\ \textit{Consumption} & C_1 & = & a_1 + c_1Y_1 \\ \textit{Exports to } R_2 & X_1tR_2 & = & \overline{X_1tR_2} \\ \textit{Exports to } R_3 & X_1tR_3 & = & \overline{X_1tR_3} \\ \textit{Imports from } R_2 & M_1fR_2 & = & m_{012} + m_{12}Y_1 \\ \textit{Imports from } R_3 & M_1fR_3 & & m_{013} + m_{13}Y_1 \end{array} \qquad (1)$$

where the four endogenous variables are: Y_1, representing income; C_1, representing private consumption; M_1fR_2, standing for imports of country 1 from country 2 and M_1fR_3, representing imports of country 1 from country 3.

The three exogenous variables are: Aut_1, domestic autonomous demand, comprising net public expenditures and private investment, and external autonomous demand, comprising: exports from country 1 to country 2 (X_1tR_2) and exports from country 1 to country 3 (X_1tR_3).

The 6 parameters assumed to be constant are: a_1, c_1, m_{012}, m_{12}, m_{013}, m_{13}, standing respectively for: autonomous private consumption, marginal propensity to consume, autonomous import demand of country 2, marginal propensity to import from country 2, autonomous import demand of country 3, marginal propensity to import from country 3. In matrix form the system can be written:

$$\begin{bmatrix} Y_1 & C_1 & M_1fR_2 & M_1fR_3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & 1 \\ -c_1 & 1 & 0 & 0 \\ -m_{12} & 0 & 1 & 0 \\ -m_{13} & 0 & 0 & 1 \end{bmatrix} = - \begin{bmatrix} Aut_1 + X_1tR_2 + X_1tR_3 \\ a_1 \\ m_{012} \\ m_{013} \end{bmatrix} \qquad (2)$$

its final form solution, where each endogenous variable is expressed in terms of exogenous variables only, is:

$$
\begin{aligned}
Y_1 &= \frac{a_1 + Aut_1 - m_{012} - m_{013} + X_1tR_2 + X_1tR_3}{1 + m_{12} + m_{13} - c_1} \\
C_1 &= \frac{a_1 + a_1m_{12} + a_1m_{13} + Aut_1c_1 - m_{012}c_1 - m_{013}c_1 + X_1tR_2c_1 + X_1tR_3c_1}{1 + m_{12} + m_{13} - c_1} \\
M_1fR_2 &= \frac{m_{012} + a_1m_{13} + Aut_1m_{12} - m_{013}m_{12} + m_{012}m_{13} - m_{012}c_1 + X_1tR_2m_{12} + X_1tR_3m_{12}}{1 + m_{12} + m_{13} - c_1} \\
M_1fR_3 &= \frac{m_{013} + m_{013}m_{12} + a_1m_{13} + Aut_1m_{13} - m_{012}m_{13} - m_{013}c_1 + X_1tR_2m_{13} + X_1tR_3m_{13}}{1 + m_{12} + m_{13} - c_1}
\end{aligned}
\qquad (3)
$$

The three multipliers of autonomous demand on country 1's incomes are thus:

$$
\begin{aligned}
\frac{\partial Y_1}{\partial Aut_1} &= M^{Y_1}_{Aut_1} = \frac{1}{1 + m_{12} + m_{13} - c_1} \\
\frac{\partial Y_1}{\partial X_1tR_2} &= M^{Y_1}_{X_1tR_2} = \frac{1}{1 + m_{12} + m_{13} - c_1} \\
\frac{\partial Y_1}{\partial X_1tR_3} &= M^{Y_1}_{X_1tR_3} = \frac{1}{1 + m_{12} + m_{13} - c_1}
\end{aligned}
\qquad (4)
$$

i.e. they are the same for domestic autonomous demand (autonomous public expenditure and private investment) and external demand (versus Region 2 and Region 3 respectively).

One can note that these multipliers are set for a "small open-economy[1]" and they differ from the "closed economy multipliers" in that, for the latter, since there are no imports, the terms m_{12} and m_{13} vanish in the denominator, whose sum $m_{12}+m_{13}$ represents the total import propensity of country 1 to import from regions 2 and 3 together.[2]

One can check that the difference between the multiplier for the "closed economy" and that for the "small open economy" is given by an expression that is always positive for reasonable values of the parameters[3]:

$$
\frac{1}{1-c_1} - \frac{1}{1+m_{12}+m_{13}-c_1} = \frac{m_{12}+m_{13}}{(1+m_{12}+m_{13}-c_1)(1-c_1)} > 0 \quad if \quad 0 < m_{12}, m_{13}, c_1 < 1 \qquad (5)
$$

For the Western Balkans region (WB6), the value of total import propensity (the numerator in the relation (5) above) is of the order of 60%. The table below gives this value estimated from the national income accounts of Eurostat for 2011, together with a calculated breakdown between that part that comes from the Western Balkans and that part that comes from the rest of the world.

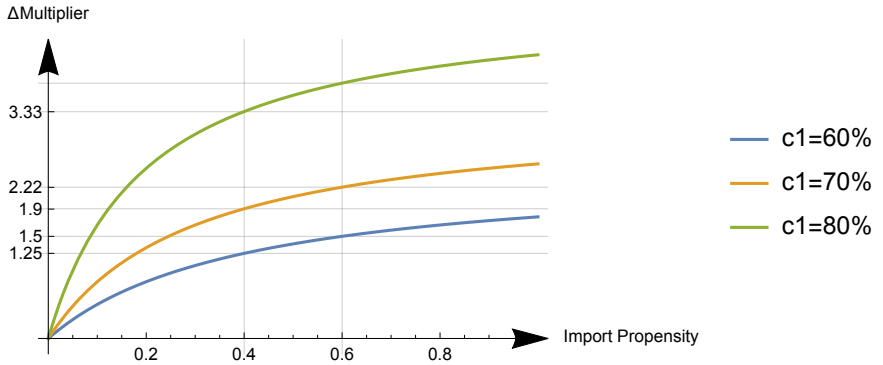
	Exports/Imports of Goods and Services in % of GDP	*Estimated portion from WB and Croatia*	*Estimated portion from rest of the World*
WB6 Imports	56.7%	15.6%	41.1%
WB6 Exports	36.1%	12.7%	23.4%
WB7 Imports	50.5%	12.1%	38.4%
WB7 Exports	37.8%	13.3%	24.4%

Source: authors' calculations based on Eurostat's online databases and Garbellini and Wirkiermann (2012)

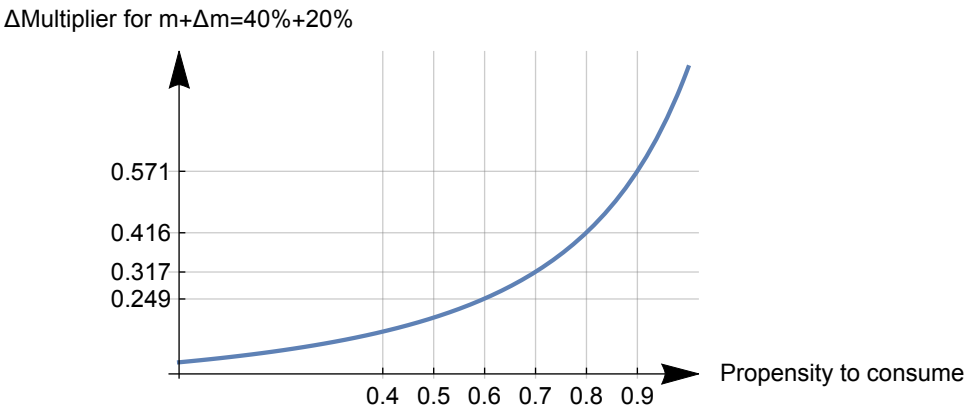
[1] See Dornbush (1980).
[2] The usual expression of the foreign trade multiplier is 1/m, corresponding to 1/(m12+m13) in the example of relation 3, see for instance Polak (1947) or Machlup (1965 [1943]). Calculated from total imports of goods and services as reported in the national accounts, this value of the multiplier is imprecise for the reason that it does not take properly into account intermediate goods; see Kennedy and Thirlwall (1979).
[3] These values are conditions for stability of the associated static equilibrium model (see Metzler, 1950).

Based on relation (5), one can see that if country 1 (say one country of the WB6) has a propensity to consume of 80%, imports 16% of its GDP from Country 2 (rest of the WB6) and another 41% from country 3 (the rest of the world), the "closed economy multiplier" is 5, whereas the "small open economy multiplier" given above in (3) is 1.3, the difference being 3.7, which is substantial[1].

This difference increases with the total import propensity from country 2 and 3 together and with the propensity to consume. The chart below illustrates how the difference between the closed and the open economy multipliers changes when the total import propensity ($m=m_{12}+m_{13}$) varies between 0 and 1 for three different values of the propensity to consume:



The chart below describes the evolution of the difference between the closed and open economy multipliers for an import propensity change from 60% to 40% as a function of the propensity to consume. It shows that the differences increase from .25 to .6 in absolute terms when import propensity goes from 60% to 90%.



The definition of the demand block in countries 2 and 3 is perfectly symmetrical with that of country 1. With the relevant country indices, the equivalent expression of relation (2) for countries 2 and 3, are given respectively in (6) and (7) for Model 2 and Model 3 respectively:

$$\begin{bmatrix} Y_2 & C_2 & M_2fR_1 & M_2fR_3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & 1 \\ -c_2 & 1 & 0 & 0 \\ -m_{21} & 0 & 1 & 0 \\ -m_{23} & 0 & 0 & 1 \end{bmatrix} = - \begin{bmatrix} Aut_2 + X_2tR_1 + X_2tR_3 \\ a_2 \\ m_{021} \\ m_{023} \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} Y_3 & C_3 & M_3fR_1 & M_3fR_2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & 1 \\ -c_3 & 1 & 0 & 0 \\ -m_{31} & 0 & 1 & 0 \\ -m_{32} & 0 & 0 & 1 \end{bmatrix} = - \begin{bmatrix} Aut_3 + X_3tR_1 + X_3tR_2 \\ a_3 \\ m_{031} \\ m_{032} \end{bmatrix} \quad (7)$$

[1] Even considering that in some of the smaller economies of the region, such as Kosovo, Montenegro and Bosnia and Herzegovina, the propensity to consume is high (90%) and that it can approximate 100% if some portion of public consumption is not considered as autonomous, import propensities are also very high and therefore the open economy multiplier is just marginally higher (1.7 rather than 1.3 in the example in text), therefore the difference with the closed economy multiplier is still high.

and the associated multipliers are:

$$\begin{aligned}
\frac{\partial Y_2}{\partial Aut_2} &= M^{Y_2}_{Aut_2} = \frac{1}{1+m_{21}+m_{23}-c_2} \\
\frac{\partial Y_2}{\partial X_2 tR_1} &= M^{Y_2}_{X_2 tR_1} = \frac{1}{1+m_{21}+m_{23}-c_2} \\
\frac{\partial Y_2}{\partial X_2 tR_3} &= M^{Y_2}_{X_2 tR_3} = \frac{1}{1+m_{21}+m_{23}-c_1}
\end{aligned} \qquad (8)$$

$$\begin{aligned}
\frac{\partial Y_3}{\partial Aut_3} &= M^{Y_3}_{Aut_1} = \frac{1}{1+m_{31}+m_{32}-c_1} \\
\frac{\partial Y_3}{\partial X_3 tR_1} &= M^{Y_3}_{X_3 tR_1} = \frac{1}{1+m_{31}+m_{32}-c_1} \\
\frac{\partial Y_3}{\partial X_3 tR_2} &= M^{Y_3}_{X_3 tR_2} = \frac{1}{1+m_{31}+m_{32}-c_1}
\end{aligned} \qquad (9)$$

One can further observe that putting the three closed economies together and treating their exports as exogenous, as done below in Model 4, is equivalent to the following system in terms of multipliers:

Model	$\left(\begin{array}{l} Y1 == AUT1 + C1 - M1fR2 - M1fR3 + X1tR2 + X1tR3 \\ C1 == a1 + pc1\ Y1 \\ M1fR2 == m012 + m12\ Y1 \\ M1fR3 == m013 + m13\ Y1 \\ Y2 == AUT2 + C2 - M2fR1 - M2fR3 + X2tR1 + X2tR3 \\ C2 == a2 + pc2\ Y2 \\ M2fR1 == m021 + m21\ Y2 \\ M2fR3 == m023 + m23\ Y2 \\ Y3 == AUT3 + C3 - M3fR1 - M3fR2 + X3tR1 + X3tR2 \\ C3 == a3 + pc3\ Y3 \\ M3fR1 == m031 + m31\ Y3 \\ M3fR2 == m032 + m32\ Y3 \end{array}\right)$	Solution	$\left(\begin{array}{l} Y1 == \frac{a1+AUT1-m012-m013+X1tR2+X1tR3}{1+m12+m13-pc1} \\ C1 == \frac{a1+a1\ m12+a1\ m13+AUT1\ pc1-m012\ pc1-m013\ pc1+pc1\ X1tR2+pc1\ X1tR3}{1+m12+m13-pc1} \\ M1fR2 == \frac{m012+a1\ m12+AUT1\ m12-m013\ m12+m012\ m13-m012\ pc1+m12\ X1tR2+m12\ X1tR3}{1+m12+m13-pc1} \\ M1fR3 == \frac{m013+m013\ m12+a1\ m13+AUT1\ m13-m012\ m13-m013\ pc1+m13\ X1tR2+m13\ X1tR3}{1+m12+m13-pc1} \\ Y2 == \frac{a2+AUT2-m021-m023+X2tR1+X2tR3}{1+m21+m23-pc2} \\ C2 == \frac{a2+a2\ m21+a2\ m23+AUT2\ pc2-m021\ pc2-m023\ pc2+pc2\ X2tR1+pc2\ X2tR3}{1+m21+m23-pc2} \\ M2fR1 == \frac{m021+a2\ m21+AUT2\ m21-m023\ m21+m021\ m23-m021\ pc2+m21\ X2tR1+m21\ X2tR3}{1+m21+m23-pc2} \\ M2fR3 == \frac{m023+m023\ m21+a2\ m23+AUT2\ m23-m021\ m23-m023\ pc2+m23\ X2tR1+m23\ X2tR3}{1+m21+m23-pc2} \\ Y3 == \frac{a3+AUT3-m031-m032+X3tR1+X3tR2}{1+m31+m32-pc3} \\ C3 == \frac{a3+a3\ m31+a3\ m32+AUT3\ pc3-m031\ pc3-m032\ pc3+pc3\ X3tR1+pc3\ X3tR2}{1+m31+m32-pc3} \\ M3fR1 == \frac{m031+a3\ m31+AUT3\ m31-m032\ m31+m031\ m32-m031\ pc3+m31\ X3tR1+m31\ X3tR2}{1+m31+m32-pc3} \\ M3fR2 == \frac{m032+m032\ m31+a3\ m32+AUT3\ m32-m031\ m32-m032\ pc3+m32\ X3tR1+m32\ X3tR2}{1+m31+m32-pc3} \end{array}\right)$

This is convenient manipulation, since Model 4, which has the same multipliers as Models 1 to 3 taken together but separately, will be the basis for endogenizing intra-regional trade in the following paragraphs.

Harrod's static foreign trade multiplier allows to define the GDP growth rate consistent with the equilibrium of the current account of the balance of payments in an independent geographical area open to external trade, which represents a dynamic extension of this multiplier, and is given by the ratio of the elasticity of exports to world income divided by the domestic income elasticity of imports. In its simplest variant, Thirlwall's law asserts that this ratio is a good predictor of actual GDP growth. Relative prices, exchange rates and capital movements can be introduced in this basic variant, generating several versions of the law. Contrary to a widespread interpretation of exports as linked to the supply-side of the economy, Thirwall interprets his laws as an empirical validation that demand matters: the fact that growth is constrained by the balance of payments in a large number of countries confirms that in these countries the generation of domestic demand is insufficient to attain the growth potential, implying that "world demand" matters as only net exports allow avoiding domestic demand shortages. Indeed, since 1979's original contribution, Thirlwall's laws were verified empirically many times, showing that they explain the growth paths followed by a large number of developing as well as by many small and medium sized advanced countries for which they were tested (Thirlwall, 2011, 2012). For Thirlwall, his results tested at the international level, apply also to regionally integrated areas linked by a fixed exchange rate, in line with Kaldor's initial insights in regional economics and consistent with the related literature on the role of regional exports in explaining the regional economic base.

Hein & Detzer (214) discuss how the balance of payments equilibrium growth rate defined by Thirlwall laws could be used as a device for policy coordination in the Euro area, where the latter is seen as a

collection of independent geographical regions bound by a common exchange rate but without a common fiscal policy, like in Model 4, where policy coordination must passively acknowledge that the aggregate demand in each area is that for the goods and services produced by its existing economic base. Obviously, as argued in the next sections, the greatest advantages of policy coordination come with active fiscal demand coordination, that can set the basis for a reallocation of the productive base in the integrated regional area.

2. Model 5: trade interdependence between country 1 and 2, independence of country 3

There is a long tradition in the analysis of trade linkages in the macroeconometric literature, starting in particular from the Link model developed by Lawrence Klein and his associates (Waelbroeck and Grinwis, 1971; Waelbroeck, 1975). In order to introduce the interdependence between areas 1 and 2 in Model 4 of the previous section, it is necessary to add relations that determine their reciprocal exports as done in Model 5 below. To these one can add a specification of exports to country 3 as a function to its import parameters as done in Model 3 and 4 above. On the contrary, for the purposes of the present analysis, it is not necessary to keep income and consumption of country 3 endogenous, therefore income of country 3 can be taken at an exogenous level Y3EXO.

```
Model 5
Y1 == AUT1 + C1 - M1fR2 - M1fR3 + X1tR2 + X1tR3
C1 == a1 + pc1 Y1
M1fR2 == m012 + m12 Y1
M1fR3 == m013 + m13 Y1
X1tR2 == M2fR1
X1tR3 == m031 + m31 Y3EXO
Y2 == AUT2 + C2 - M2fR1 - M2fR3 + X2tR1 + X2tR3
C2 == a2 + pc2 Y2
M2fR1 == m021 + m21 Y2
M2fR3 == m023 + m23 Y2
X2tR1 == M1fR2
X2tR3 == m032 + m32 Y3EXO
Y3 == Y3EXO
```

The full solution of Model 5 is given in the Annex. One can verify that the multipliers of country 1 and country 2 incomes with respect to the three exogenous variables Aut_1, Aut_2 and Y3EXO are given by relations (10) and (11) below.

$$\begin{aligned}
\frac{\partial Y_1}{\partial Aut_1} &= M^{Y_1}_{Aut_1} = \frac{1+m_{21}+m_{23}-c_2}{m_{12}(1+m_{23}-c_2)+(1+m_{13}-c_1)(1+m_{21}+m_{23}-c_2)} \\
\frac{\partial Y_1}{\partial Aut_2} &= M^{Y_1}_{Aut_2} = \frac{m_{21}}{m_{12}(1+m_{23}-c_2)+(1+m_{13}-c_1)(1+m_{21}+m_{23}-c_2)} \\
\frac{\partial Y_1}{\partial Y3EXO} &= M^{Y_1}_{Y3EXO} = \frac{m_{21}m_{32}+m_{31}(1+m_{21}+m_{23}-c_2)}{m_{12}(1+m_{23}-c_2)+(1+m_{13}-c_1)(1+m_{21}+m_{23}-c_2)}
\end{aligned} \tag{10}$$

$$\begin{aligned}
\frac{\partial Y_2}{\partial Aut_1} &= M^{Y_1}_{Aut_1} = \frac{1+m_{21}+m_{23}-c_2}{m_{12}(1+m_{23}-c_2)+(1+m_{13}-c_1)(1+m_{21}+m_{23}-c_2)} \\
\frac{\partial Y_2}{\partial Aut_2} &= M^{Y_1}_{Aut_2} = \frac{m_{21}}{m_{12}(1+m_{23}-c_2)+(1+m_{13}-c_1)(1+m_{21}+m_{23}-c_2)} \\
\frac{\partial Y_2}{\partial Y3EXO} &= M^{Y_1}_{Y3EXO} = \frac{m_{21}m_{32}+m_{31}(1+m_{21}+m_{23}-c_2)}{m_{12}(1+m_{23}-c_2)+(1+m_{13}-c_1)(1+m_{21}+m_{23}-c_2)}
\end{aligned} \tag{11}$$

One can also check there that, for Model 5, the multiplier of autonomous demand in country 1 (Aut_1) on Y_1 given by relation (10) is higher than the same multiplier for Model 1, given by relation (4). Indeed developing from the assumption that the multiplier in (10) is lower than that in (4), for reasonable values

of the parameters such as $m_{12} \geq 0$, $m_{21} \geq 0$, $m_{13} \geq 0$, $m_{23} \geq 0$, $c_1 \geq 0$, $c_2 \geq 0$, $c_1 \leq 1$, $c_2 \leq 1$, one finds a contradiction.

$$\textit{Putting:} \quad \frac{1+m_{21}+m_{23}-c_2}{m_{12}(1+m_{23}-c_2)+(1+m_{13}-c_1)(1+m_{21}+m_{23}-c_2)} < \frac{1}{1+m_{12}+m_{13}-c_1} \quad \textit{gives:}$$

$$\frac{(1+m_{21}+m_{23}-c_2)(1+m_{12}+m_{13}-c_1)}{[m_{12}(1+m_{23}-c_2)+(1+m_{13}-c_1)(1+m_{21}+m_{23}-c_2)](1+m_{12}+m_{13}-c_1)} < \frac{[m_{12}(1+m_{23}-c_2)+(1+m_{13}-c_1)(1+m_{21}+m_{23}-c_2)]}{[m_{12}(1+m_{23}-c_2)+(1+m_{13}-c_1)(1+m_{21}+m_{23}-c_2)](1+m_{12}+m_{13}-c_1)}$$

$$\textit{or:} \quad (1+m_{21}+m_{23}-c_2)(1+m_{12}+m_{13}-c_1-1-m_{13}+c_1) < m_{12}(1+m_{23}-c_2) \quad \textit{The latter brings to a contradiction because:}$$

$$(1+m_{21}+m_{23}-c_2)-(1+m_{23}-c_2) = m_{21} < 0 \quad \textit{which is contrary to what was assumed in the beginning.}$$

Similarly one can also verify that the multiplier of Y_2 with respect to autonomous demand in country 2 in (11) is higher than the relevant multiplier in (6). One can thus conclude that, once repercussion effects between country 1 and country 2 are taken into account in a regionally integrated area where two countries fix independently their level of autonomous demand, the effect of an autonomous domestic increase in demand is higher because the latter increases also imports from country 2, which increases its income, and therefore its imports from 1.

As a special case of Model 5 one can check that for a two country model where output is considered as the endogenous variable rather than income, the multipliers (8) and (9) with no repercussion reduce to those presented in Brems (1956), therefore model 5 generalizes his model.

Coming back on the issue of policy coordination, one can see in the charts page 6, that, if by policy coordination, one can endogeneize the portion of imports that comes from the regionally integrated area (Western Balkan's intra-trade), thus decreasing import propensity from 60% to 40%, the difference between the closed and open economy multiplier would decrease by an amount that, depending from the propensity to consume, varies between 11% and 20%: from 3.75 to 3.5 if c1=80%, from 2.22 to 1.9 if c1=70% and from 1.5 to 1.25 if c1=60%. The reduction in the difference between the closed and open multipliers corresponds to an increase in the open multiplier itself, which is obviously higher in % terms: going from 13% to 25% depending from the propensity to consume in the examples above. This increase in the open economy multiplier (the actual multiplier, given prevailing trade circumstances) represents the dividend from policy cooperation. It is clearly more important for regionally integrated areas (where m_{12} is relatively higher with respect to m_{13}). Given the likely value of the parameters in the Western Balkans, the percentage increase in the value of the open economy multiplier is of the order of the share of intra-regional trade. This idea is further developed in the next section.

3. Model 6: trade interdependence between country 1 and 2, independence of country 3 and policy cooperation between country 1 and 2

Once it is understood that policy decisions taken in one country affect the situation in all countries that are linked with it through trade, it is logical to start exploring the possibilities for policy cooperation. This can be illustrated by taking countries 1 and 2 as parts of an integrated regional area, as it is done with Model 6, which is the same as Model 5, except that a new endogenous variable is added: the total income of regions 1 and 2, which is just given by their sum $Y_{12}=Y_1+Y_2$. For studying the effects of autonomous shocks, taking country 1 and country 2 as an aggregate is equivalent to considering them as a single country.

Model 6 is given by:

```
Model
{ Y1 == AUT1 + C1 - M1fR2 - M1fR3 + X1tR2 + X1tR3
  C1 == a1 + pc1 Y1
  M1fR2 == m012 + m12 Y1
  M1fR3 == m013 + m13 Y1
  X1tR2 == M2fR1
  X1tR3 == m031 + m31 Y3EXO
  Y2 == AUT2 + C2 - M2fR1 - M2fR3 + X2tR1 + X2tR3
  C2 == a2 + pc2 Y2
  M2fR1 == m021 + m21 Y2
  M2fR3 == m023 + m23 Y2
  X2tR1 == M1fR2
  X2tR3 == m032 + m32 Y3EXO
  Y3 == Y3EXO
  Y12 == Y1 + Y2 }
```

One can verify that for this model the final form solution is that given in Annex 2, which implies the following multipliers for total income of countries 1 and 2 (Y_{12}):

$$\begin{aligned}
\frac{\partial Y_{12}}{\partial Aut_1} &= M_{Aut_1}^{Y_{12}} = \frac{1+m_{21}+m_{21}+m_{23}-c_2}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)} \\
\frac{\partial Y_{12}}{\partial Aut_2} &= M_{Aut_2}^{Y_{12}} = \frac{1+m_{12}+m_{13}+m_{21}-c_1}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)} \\
\frac{\partial Y_{12}}{\partial Y3EXO} &= M_{Y3EXO}^{Y_{12}} = \frac{m_{32}\left(1+m_{12}+m_{13}+m_{21}-c_1\right)+m_{31}\left(1+m_{12}+m_{21}+m_{23}-c_2\right)}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)}
\end{aligned} \tag{12}$$

Since the denominators in these multipliers are the same as those in (10) and (11) it is relatively simple to check that the three multipliers in (12) are higher than either (10) and (11), as in fact they represent their sum as shown in the table below:

	Y_1	Y_2	Y_{12}
Aut_1	$\frac{1+m_{21}+m_{23}-c_2}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)}$	$\frac{m_{12}}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)}$	$\frac{1+m_{21}+m_{21}+m_{23}-c_2}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)}$
Aut_2	$\frac{m_{21}}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)}$	$\frac{1+m_{12}+m_{13}-c_1}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)}$	$\frac{1+m_{12}+m_{13}+m_{21}-c_1}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)}$
$Y3EXO$	$\frac{m_{21}m_{32}+m_{31}\left(1+m_{21}+m_{23}-c_2\right)}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)}$	$\frac{m_{12}\left(m_{31}+m_{32}\right)+m_{32}\left(1+m_{13}-c_2\right)}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)}$	$\frac{m_{32}\left(1+m_{12}+m_{13}+m_{21}-c_1\right)+m_{31}\left(1+m_{12}+m_{21}+m_{23}-c_2\right)}{m_{12}\left(1+m_{23}-c_2\right)+\left(1+m_{13}-c_1\right)\left(1+m_{21}+m_{23}-c_2\right)}$

(13)

It is thus clear that the total impact of an increase of autonomous demand in country 1 has higher effects on the total of incomes of country 1 and 2 than on the incomes of each of the countries.

An example can serve to fix the ideas. The following values for the parameters are relatively realistic for the larger Western Balkans countries and simplify the argument[1]:

Autonomous import demand	m_{012} =0	m_{013} = 0	m_{021} = 0	m_{023} = 0	m_{031} = 0	m_{032} = 0
Import propensities	m_{12} = 0.15	m_{13} = 0.35	m_{21} = 0.15	m_{23} = 0.35	m_{31} = 0.4	m_{32} = 0.4
Consumption coefficients	c_1 = 0.65	c_2 = 0.7	a_1 = 0	a_2 = 0	-	-

Under these parameters, the multipliers are as follows:

	Y1	Y2	Y12	Y1+Y2
AUT1	1.21673	0.228137	1.44487	1.44487
AUT2	0.228137	1.29278	1.52091	1.52091
Y3EXO	0.577947	0.608365	1.18631	1.18631

[1] The value for m_{31} implies that the rest of the world is treated as a "small country", of the size of the Western Balkans, which can be done by a change of unit. In that case the value of 0.4 is acceptable.

One can observe that for the same parameters the value of the multipliers in Model 1 to 4 are respectively 1.176 for country 1 and 1.25 for country 2, therefore, with reference to the discussion at the end of the previous section, it is apparent that by endogenizing exports going from Model 4 to Model 5 the multiplier increases from 1.17 to 1.21 in country 1 and from 1.25 to 1.29 in country 2. However this is only part of the story, because in fact the increase in autonomous demand in country 1 also has effects on income of country 2, for which the multiplier is .22. Thus the total effect on country 1 and country 2 together of an increase in autonomous demand in country 1 is 1.44, which is significantly higher than 1.176 (+22%). The same is true for country 2: from 1.25 in model 2, the multiplier of an autonomous demand shock in country 2 becomes 1.29 and the total effect on country 1 and 2 is 1.52 (+22%). It is thus clear that by giving coordinated demand shock country 1 and 2 can substantially increase their joint income. The latter is given by the sum of the two first multipliers in the last column of the table, which is in this case: 2.96 that divided by 2 gives 1.48. To obtain the same effect by an uncoordinated autonomous demand shock, would have required 1.26 in country 1 and 1.18 in country 2, against a shock of 1 in both countries when they value policy coordination.

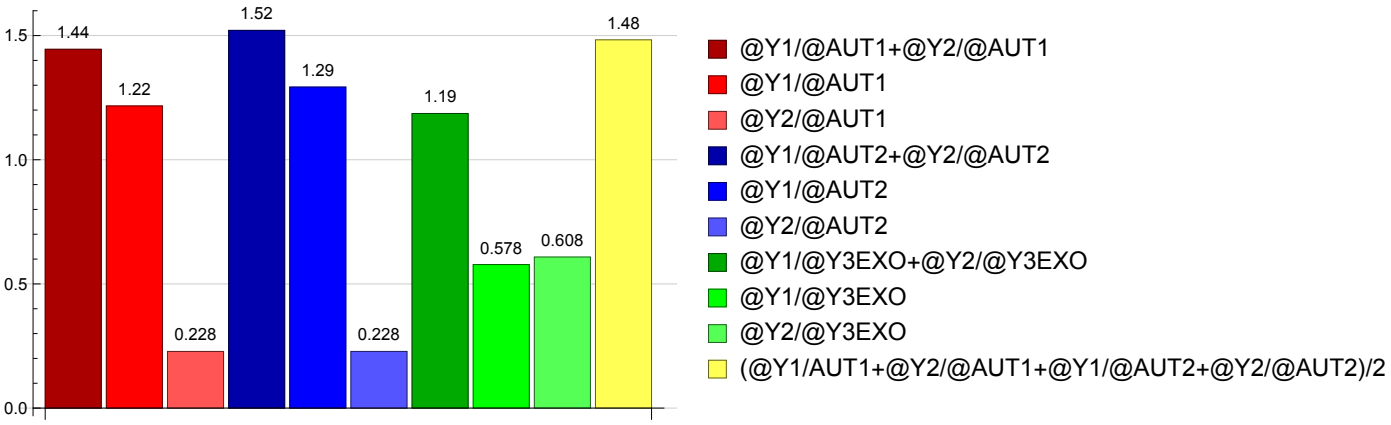
To conclude, a word can also be said about competitiveness policies, which aim at improving the domestic cost conditions and thus improving exports to the rest of the world[1]. The multipliers in the table above, depicted in the chart in the next page below, show that if exogenous demand for country 1 and country 2 regions exports (imports for region 3) would increase by an amount corresponding to a unitary shock on autonomous demand in country 3, then the multiplier effect on countries 1 and 2 is of the same order, although significantly lower (1.19 against 1.48), to that of an unitary increase in domestic demand in both countries[2].

However, one should take into consideration that often competitiveness policies are based on wage compression and therefore imply first a negative effect on domestic income, like it was shown in recent developments in the European Union. Therefore, when one compares the effects the multiplier effect of a unitary increase in domestic or external demand, the net effect of competitiveness policies is not given by the unit labour cost shock multiplied by the elasticity of exports to country 3, but by a lower multiplier figure that takes into account the reduction of domestic output following wage deflation, which could be substantial. In fact, it is now generally acknowledged that growth is wage rather than profit led (see for instance Lavoie and Stockhammer, 2012 and 2013) and therefore an intelligent policy of wage increases is supportive to growth, whereas wage deflation has cumulative negative effects on economic activity[3].

[1] Examining a large sample of Belgian exporting firms, Decramer, Fuss and Konings (2014) find an elasticity of exports to unit labour costs, the most common measure retained for cost competitiveness, comprised between -0.2 and -0.4, i.e. rather low. It means that if unit labour costs were decreased by 10%, exports would increase by 4%.

[2] As mentioned before, country 3 is assumed in this example to be of the same size of country 1 and 2, representing the Western Balkans, in order to retain an import propensity of 40%. That is equivalent to assuming a different unit for Y_3 and for Y_1 and Y_2. However, the argument is not affected by this change of unit. Under these assumptions, a unitary increase in demand of country 3 corresponds to an absolute increase in exports of country 1 or 2 by 1 unit, which, with an export to GDP ratio of 40% corresponds to a gain of 6% in unit labour costs.

[3] The argument for wage-led growth can also be appreciated with reference to Kalecki's interpretation of the multiplier, which sees it as expressing distributive shares, with the denominator equal to the profit share of income in the simplest case of a closed economy where workers do not save. When the economy is opened to foreign trade, things get more complicated, but the various parameters at play make it relatively plausible that the positive effect on export demand of a decrease in wages is more than compensated by the negative effect on domestic demand through a deterioration of the distribution between wages and profits to the detriment of wages (see for instance Laski and Walther, 2015). Therefore in the short-term, when private investment is constrained by lack of demand and fiscal space considerations limit public investment, productivity cannot increase for lack of investment. The only way to obtain gains in unit labour costs is then to reduce wages, which, as argued above, is likely to provoke a compensating reduction in domestic demand. This implies that interregional coordination of demand through the foreign trade multiplier is a more effective way to restore growth that wage deflation as it allows to



On the contrary, if it would be possible to obtain a significant cost reduction by a devaluation of the exchange rate, then this multiplier effect of exogenous demand would be achieved at no cost in terms of domestic deflation, as it is the case for instance for the exchange rate policy followed by China after its entry in the World Trade Organization, whose effect on the Euro area are now mitigated by the appreciation of the dollar. However, apart for the case when exchange rate control still exist, like in China, it is not possible to keep artificially a low exchange rate in a sustainable way, therefore sooner or later in a country that runs a current account surplus there would be an appreciation of the exchange rate.

To come back on the discussion of Thirlwall laws at the end of sections 1, and on policy coordination at the end of section 2, for a regionally integrated area, an alternative to a passive policy of demand coordination aiming at preserving balance of payments equilibrium as the one discussed by Hein and Detzer (2014), would be an active policy of demand coordination exploiting regional trade multipliers as that suggested in the present contribution. Such an alternative is all the more rational, if not necessary, given that in a regionally integrated area, by definition, not only goods and services, but also factors of production, cross borders through trade, and, as recent developments confirm, not always in a balancing way.

Conclusion

The analysis presented in this paper illustrates and extends an argument for policy coordination put forward in an elegant example made by Muet (2004, pp. 128-132) for the case of two countries with the same import elasticity to income. In the special case examined by Muet, the fiscal policy multiplier with economic policy coordination is the same as the closed economy multiplier, which as argued above, is of course substantially higher than the open economy multiplier without policy coordination.

Based on a discussion of a three countries example this contribution shows that in more general cases, the fiscal policy multiplier with coordination is less than the closed economy multiplier but higher than the multiplier without policy coordination. In other words the difference between the multiplier with and

achieve productivity gains once private investment responds to the improvement in demand conditions, which on the contrary is unlikely to ever occur under wage deflation.

without policy coordination is necessarily positive, this difference being higher the higher the share of intra-regional trade in the countries.

Coordination of investment policy in regionally integrated areas, as for instance the one promoted under the *Western Balkans Investment Framework* [1], is thus beneficial. In particular, it can be achieved by focusing investment on a network of regional investment projects which can be financed in all countries at the same time and contribute to improve supply conditions. Some developments of the above analysis can be found in Cingolani, Garbellini and Wirkiermann (2012 and 2014), in particular on the question of a rational sharing of the cost and benefits of policy coordination. Such active policy coordination towards growth and employment creation represents a possible alternative to a passive policy of demand coordination aiming at preserving balance of payments equilibrium through cost competitiveness.

[1] see: www.wbif.eu.

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Annex: final form solutions to models 5 and 6

Model 5:

Solution

$$Y1 == \frac{-m012-m013+m021+m031+a2\,m21+AUT2\,m21-m013\,m21-m023\,m21+m031\,m21+m032\,m21-m012\,m23-m013\,m23+m021\,m23+m031\,m23+a1\,(1+m21+m23-pc2)+AUT1\,(1+m21+m23-pc2)+m012\,pc2+m013\,pc2-m021\,pc2-m031\,pc2+(m21\,m32+m31\,(1+m21+m23-pc2))\,Y3EXO}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$C1 == \frac{a1\,(m12\,(1+m23-pc2)+(1+m13)\,(1+m21+m23-pc2))+pc1\,(m021+m031+(a2+AUT2)\,m21-m013\,(1+m21)-(m013-m021-m031)\,(m23-pc2)+AUT1\,(1+m21+m23-pc2)+m012\,(-1-m23+pc2)+m31\,Y3EXO+(m23\,m31+m21\,m32-m31\,pc2)\,Y3EXO+m21\,(-m023+m031+m032+m31\,Y3EXO))}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$M1fR2 == \frac{m012\,(1+m13-pc1)\,(1+m21+m23-pc2)+m12\,(-m013+m021+m031+a2\,m21+AUT2\,m21-m013\,m21-m023\,m21+m031\,m21+m032\,m21-m013\,m23+m021\,m23+m031\,m23+a1\,(1+m21+m23-pc2)+AUT1\,(1+m21+m23-pc2)+m013\,pc2-m021\,pc2-m031\,pc2+(m21\,m32+m31\,(1+m21+m23-pc2))\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$M1fR3 == \frac{m013\,(m12\,(1+m23-pc2)-(-1+pc1)\,(1+m21+m23-pc2))+m13\,(-m012+m021+m031+a2\,m21+AUT2\,m21-m023\,m21+m031\,m21+m032\,m21-m012\,m23+m021\,m23+m031\,m23+a1\,(1+m21+m23-pc2)+AUT1\,(1+m21+m23-pc2)+m012\,pc2-m021\,pc2-m031\,pc2+(m21\,m32+m31\,(1+m21+m23-pc2))\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$X1tR2 == \frac{m021\,(1+m12+m13-pc1)\,(1+m23-pc2)+m21\,(m012-m023+m032+(a1+AUT1-m013)\,m12-m023\,m12+m031\,m12+m032\,m12+m012\,m13-m023\,m13+m032\,m13+a2\,(1+m12+m13-pc1)+AUT2\,(1+m12+m13-pc1)-m012\,pc1+m023\,pc1-m032\,pc1+(m32+m12\,(m31+m32))\,Y3EXO+m32\,(m13-pc1)\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$m031 + m31\,Y3EXO == X1tR3$$

$$Y2 == \frac{m012-m021-m023+m032+(a1+AUT1)\,m12-m013\,m12-m023\,m12+m031\,m12+m032\,m12-m023\,m13+m032\,m13+(m012-m021)\,(m13-pc1)+a2\,(1+m12+m13-pc1)+AUT2\,(1+m12+m13-pc1)+m023\,pc1-m032\,pc1+(m12\,(m31+m32)+m32\,(1+m13-pc1))\,Y3EXO}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$C2 == \frac{a2\,(m12\,(1+m23)+(1+m21+m23)\,(1+m13-pc1))+pc2\,(-m021+m032+(a1+AUT1-m013)\,m12+(m031+m032)\,m12-m023\,(1+m12)+m012\,(1+m13-pc1)+AUT2\,(1+m12+m13-pc1)+(m32+m12\,(m31+m32))\,Y3EXO-(m13-pc1)\,(m021+m023-m032-m32\,Y3EXO))}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$M2fR1 == \frac{m021\,(1+m12+m13-pc1)\,(1+m23-pc2)+m21\,(m012-m023+m032+(a1+AUT1-m013)\,m12-m023\,m12+m031\,m12+m032\,m12+m012\,m13-m023\,m13+m032\,m13+a2\,(1+m12+m13-pc1)+AUT2\,(1+m12+m13-pc1)-m012\,pc1+m023\,pc1-m032\,pc1+(m32+m12\,(m31+m32))\,Y3EXO+m32\,(m13-pc1)\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$M2fR3 == \frac{m023\,(m12+(1+m13-pc1)\,(1+m21-pc2)-m12\,pc2)+m23\,(m012-m021+m032+(a1+AUT1)\,m12-m013\,m12+m031\,m12+m032\,m12+m032\,m13+(m012-m021)\,(m13-pc1)+a2\,(1+m12+m13-pc1)+AUT2\,(1+m12+m13-pc1)-m032\,pc1+(m12\,(m31+m32)+m32\,(1+m13-pc1))\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$X2tR1 == \frac{m012\,(1+m13-pc1)\,(1+m21+m23-pc2)+m12\,(-m013+m021+m031+a2\,m21+AUT2\,m21-m013\,m21-m023\,m21+m031\,m21+m032\,m21-m013\,m23+m021\,m23+m031\,m23+a1\,(1+m21+m23-pc2)+AUT1\,(1+m21+m23-pc2)+m013\,pc2-m021\,pc2-m031\,pc2+(m21\,m32+m31\,(1+m21+m23-pc2))\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$m032 + m32\,Y3EXO == X2tR3$$

$$Y3 == Y3EXO$$

Model 6:

Solution

$$Y1 == \frac{-m012-m013+m021+m031+a2\,m21+AUT2\,m21-m013\,m21-m023\,m21+m031\,m21+m032\,m21-m012\,m23-m013\,m23+m021\,m23+m031\,m23+a1\,(1+m21+m23-pc2)+AUT1\,(1+m21+m23-pc2)+m012\,pc2+m013\,pc2-m021\,pc2-m031\,pc2+(m21\,m32+m31\,(1+m21+m23-pc2))\,Y3EXO}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$C1 == \frac{a1\,(m12\,(1+m23-pc2)+(1+m13)\,(1+m21+m23-pc2))+pc1\,(m021+m031+(a2+AUT2)\,m21-m013\,(1+m21)-(m013-m021-m031)\,(m23-pc2)+AUT1\,(1+m21+m23-pc2)+m012\,(-1-m23+pc2)+m31\,Y3EXO+(m23\,m31+m21\,m32-m31\,pc2)\,Y3EXO+m21\,(-m023+m031+m032+m31\,Y3EXO))}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$M1fR2 == \frac{m012\,(1+m13-pc1)\,(1+m21+m23-pc2)+m12\,(-m013+m021+m031+a2\,m21+AUT2\,m21-m013\,m21-m023\,m21+m031\,m21+m032\,m21-m013\,m23+m021\,m23+m031\,m23+a1\,(1+m21+m23-pc2)+AUT1\,(1+m21+m23-pc2)+m013\,pc2-m021\,pc2-m031\,pc2+(m21\,m32+m31\,(1+m21+m23-pc2))\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$M1fR3 == \frac{m013\,(m12\,(1+m23-pc2)-(-1+pc1)\,(1+m21+m23-pc2))+m13\,(-m012+m021+m031+a2\,m21+AUT2\,m21-m023\,m21+m031\,m21+m032\,m21-m012\,m23+m021\,m23+m031\,m23+a1\,(1+m21+m23-pc2)+AUT1\,(1+m21+m23-pc2)+m012\,pc2-m021\,pc2-m031\,pc2+(m21\,m32+m31\,(1+m21+m23-pc2))\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$X1tR2 == \frac{m021\,(1+m12+m13-pc1)\,(1+m23-pc2)+m21\,(m012-m023+m032+(a1+AUT1-m013)\,m12-m023\,m12+m031\,m12+m032\,m12+m012\,m13-m023\,m13+m032\,m13+a2\,(1+m12+m13-pc1)+AUT2\,(1+m12+m13-pc1)-m012\,pc1+m023\,pc1-m032\,pc1+(m32+m12\,(m31+m32))\,Y3EXO+m32\,(m13-pc1)\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$m031 + m31\,Y3EXO == X1tR3$$

$$Y2 == \frac{m012-m021-m023+m032+(a1+AUT1)\,m12-m013\,m12-m023\,m12+m031\,m12+m032\,m12-m023\,m13+m032\,m13+(m012-m021)\,(m13-pc1)+a2\,(1+m12+m13-pc1)+AUT2\,(1+m12+m13-pc1)+m023\,pc1-m032\,pc1+(m12\,(m31+m32)+m32\,(1+m13-pc1))\,Y3EXO}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$C2 == \frac{a2\,(m12\,(1+m23)+(1+m21+m23)\,(1+m13-pc1))+pc2\,(-m021+m032+(a1+AUT1-m013)\,m12+(m031+m032)\,m12-m023\,(1+m12)+m012\,(1+m13-pc1)+AUT2\,(1+m12+m13-pc1)+(m32+m12\,(m31+m32))\,Y3EXO-(m13-pc1)\,(m021+m023-m032-m32\,Y3EXO))}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$M2fR1 == \frac{m021\,(1+m12+m13-pc1)\,(1+m23-pc2)+m21\,(m012-m023+m032+(a1+AUT1-m013)\,m12-m023\,m12+m031\,m12+m032\,m12+m012\,m13-m023\,m13+m032\,m13+a2\,(1+m12+m13-pc1)+AUT2\,(1+m12+m13-pc1)-m012\,pc1+m023\,pc1-m032\,pc1+(m32+m12\,(m31+m32))\,Y3EXO+m32\,(m13-pc1)\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$M2fR3 == \frac{m023\,(m12+(1+m13-pc1)\,(1+m21-pc2)-m12\,pc2)+m23\,(m012-m021+m032+(a1+AUT1)\,m12-m013\,m12+m031\,m12+m032\,m12+m032\,m13+(m012-m021)\,(m13-pc1)+a2\,(1+m12+m13-pc1)+AUT2\,(1+m12+m13-pc1)-m032\,pc1+(m12\,(m31+m32)+m32\,(1+m13-pc1))\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$X2tR1 == \frac{m012\,(1+m13-pc1)\,(1+m21+m23-pc2)+m12\,(-m013+m021+m031+a2\,m21+AUT2\,m21-m013\,m21-m023\,m21+m031\,m21+m032\,m21-m013\,m23+m021\,m23+m031\,m23+a1\,(1+m21+m23-pc2)+AUT1\,(1+m21+m23-pc2)+m013\,pc2-m021\,pc2-m031\,pc2+(m21\,m32+m31\,(1+m21+m23-pc2))\,Y3EXO)}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$

$$m032 + m32\,Y3EXO == X2tR3$$

$$Y3 == Y3EXO$$

$$Y12 == \frac{a2+AUT1+AUT2-m013-m023+m031+m032+a2\,m12+AUT1\,m12+AUT2\,m12-m013\,m12-m023\,m12+m031\,m12+m032\,m12+a2\,m13+AUT2\,m13+m012\,m13-m021\,m13-m023\,m13+m032\,m13+a2\,m21+AUT1\,m21+AUT2\,m21-m013\,m21-m023\,m21+m031\,m21+m032\,m21+AUT1\,m23-m012\,m23-m013\,m23+m021\,m23+m031\,m23-a2\,pc1-AUT2\,pc1-m012\,pc1+m021\,pc1+m023\,pc1-m032\,pc1+a1\,(1+m12+m21+m23-pc2)-AUT1\,pc2+m012\,pc2+m013\,pc2-m021\,pc2-m031\,pc2+(m32\,(1+m12+m13+m21-pc1)+m31\,(1+m12+m21+m23-pc2))\,Y3EXO}{m12\,(1+m23-pc2)+(1+m13-pc1)\,(1+m21+m23-pc2)}$$