Modelling Cumulative Capital Drawdowns resulting from EIB's investments in Private Equity Funds

European Investment Bank Capstone

The Team

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Expected Outcome

Introduction to the project

Expected Outcome

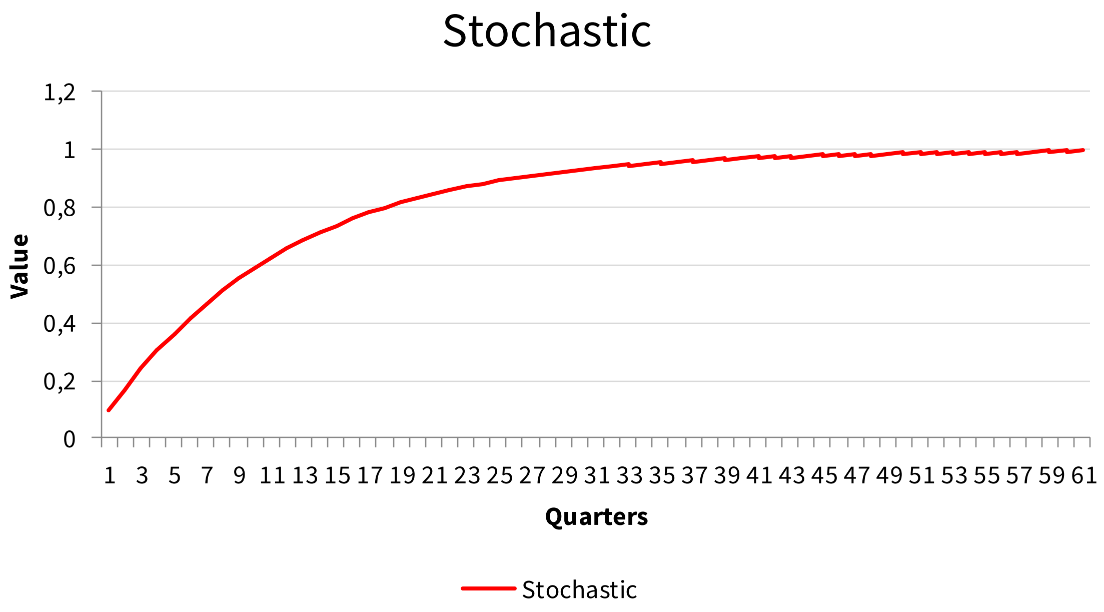
The goal of the project is the **estimation of the cumulative capital drawdowns** for a sample of **EIB's private equity fund investments.**

Relevance

Main risks during investment period:

Liquidity risk

Uncertainty on the timing and amount of drawdowns from the funds that had been committed



Expected Outcome

Introduction to the equity-type co-investments of the EIB & EIF

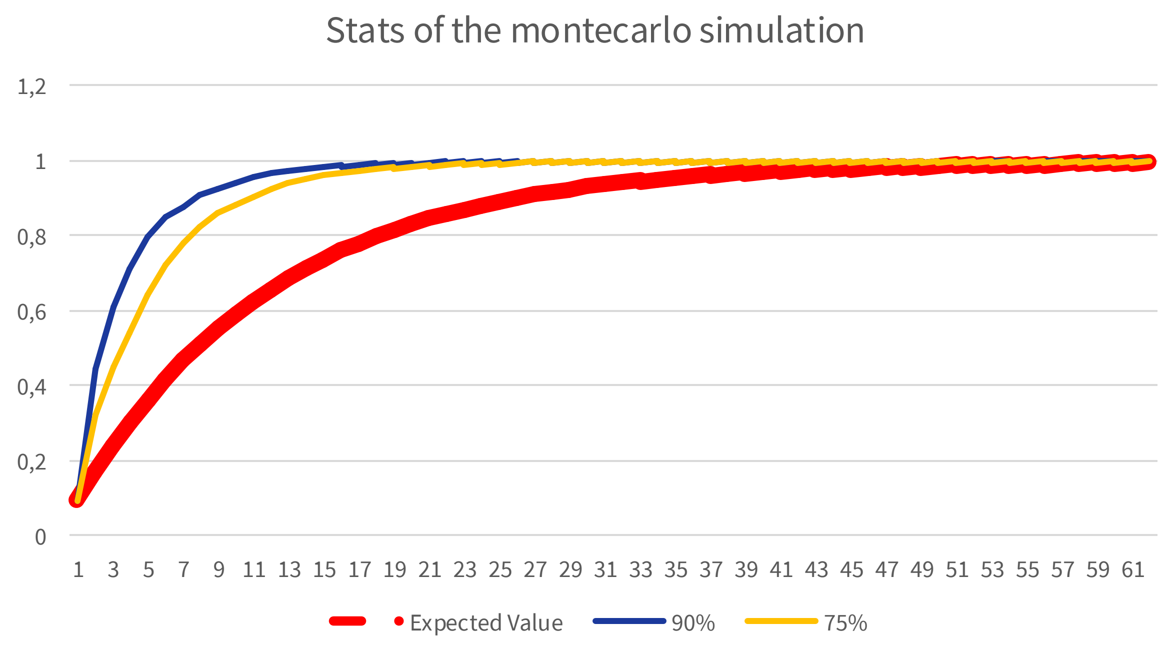
Expected Outcome

The goal of the project is the **estimation of the cumulative capital drawdowns** for a sample of **EIB's private equity fund investments.**

Models & Estimations

Yale Model
Takahashi & Alexander, 2001

Stochastic Model
Buchner, Kaserer, & Wagner, 2010



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Data Curation & Standardization

Subtracted +35% of the funds to ensure accuracy through model calibration

619	1972 to 2023	22	9,464
Funds	Time Span	Variables	Nº drawdowns

Assumptions

Assumption 1:

Commited capital = Total Disbursed Capital

Assumption 2:

Investment period starts at signature date

Data Curation & Standardization

Subtracted +35% of the funds to ensure accuracy through model calibration

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Adjustment 1

Removal of funds containing drawdowns with a closed contract type.

107

Removed funds (17.29%)

Adjustment 2

Removal of funds with signature date before 1990 and from 2020 onwards.

83

Removed funds (13.41%)

Adjustment 3.

Removal of 1 fund with no committed/disbursed capital.

1

Removed fund

Cohorts Definition

Divided data into 2 different cohorts → Ljungqvist

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Region. EU and Non-EU

The initial dataset organized funds according to their respective countries or regions of investment, resulting in 93 different cohorts. To simplify the data, we chose to group countries into broader regions, resulting in just two cohorts: EU and non-EU .

397 Funds	243 EU → 61%
	154 Non-EU → 39%

Fund Size. Small and Large

To assess the influence of fund size on drawdown patterns, we segmented the data into four cohorts based on quartiles.

However, due to the similarities in capital drawdown between small and medium funds, and between large and extremely large funds, we combined them into two groups. Small funds now consist of quartiles 1, 2 and 3 while large funds consist of quartiles 4 and all outliers.

397 Funds	284 Small → 72%
	113 Large → 28%

Data Curation & Standardization

Final Dataset contained 397 useful

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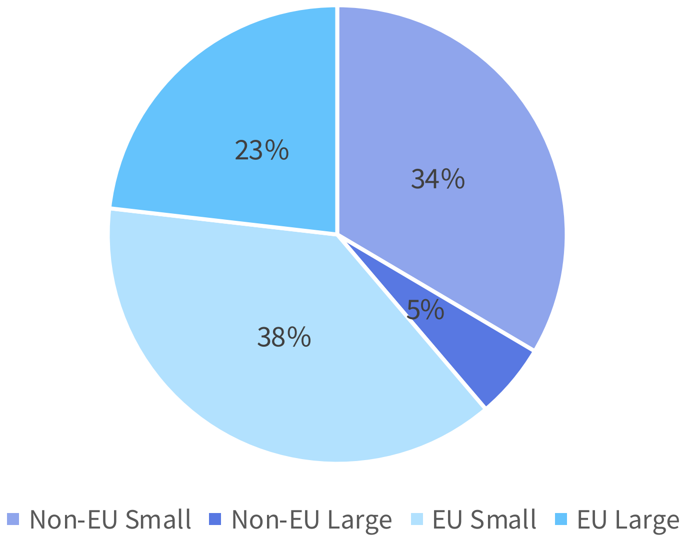
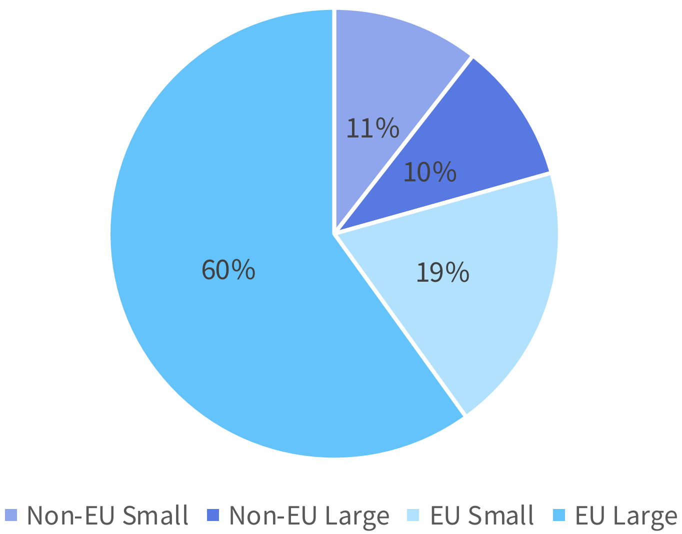
Final Dataset

Funds



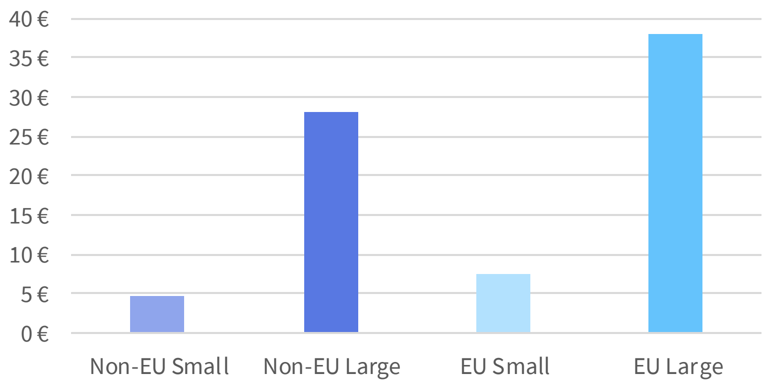
397
Useful funds

Disbursed Capital



5,824,111,348 €
Disbursed Capital

Average disbursed capital/fund (millions)



243 EU → 61%

154 Non-EU → 39%

284 Small → 72%

113 Large → 28%

Data Curation & Standardization

Final Dataset contained 397 useful funds with an average of 23.89 drawdowns/fund

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Standarization

To estimate model parameters from the observable capital drawdowns of the sample funds at equidistant time points.

To ensure comparability among funds of varying sizes, the capital drawdowns of all j = 1,...,N sample funds are initially standardized based on each fund's total disburtsment.

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The model

- Due to the nature, capital contributions (i.e. drawdowns) are heavily concentrated in the initial life of a fund, and marginally diminish as time passes. It is dependent on the Rate of Contribution (i.e. drawdown rate) of the undrawn capital:

$$dD_t = \delta_t(C - D_t)$$

- The key part of the model is the rate of contribution, which we will calculate the average of all the fund's capital contribution relative to the undrawdown rate.

$$\delta_{k,j} = \frac{d_{k,j}}{U_{k-1,j}} \qquad \hat{\delta}_k = \frac{1}{N}\sum_{j=1}^{N} \delta_{k,j}$$

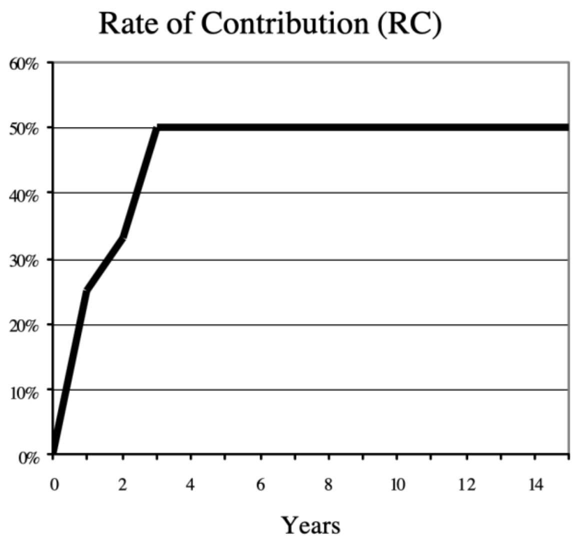
The Yale Model

Dean Takahashi - Seth Alexander

The model

- The nature of the rate of contribution, allows for a preliminary understanding of the change over time as see in the graph.

- This Concave relationship will likely be consistent amongst the funds, however, its concavity must be determined by the explanatory variables

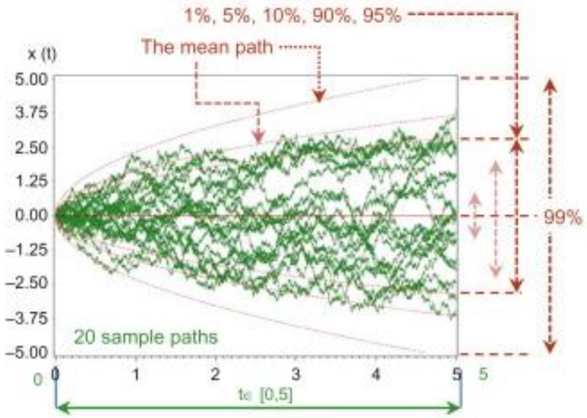


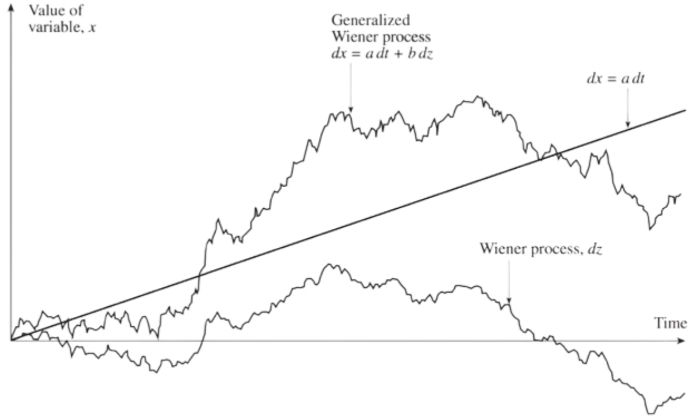
The Yale models provide a base for comparison to our Stochastic model. Its simplicity is its strength as it can be used as a reference point to determine level of improvement in accuracy for the Stochastic model

The Stochastic Model

Axel Buchner - Niklas F. Wagner - Christoph Kaserer

Stochastic Process (Brownian Motion)





A stochastic process (or also called random process) is a mathematical object usually defined as a sequence of random variables in a probability space.

There are different types, but the model focuses on one: **Brownian Motion**.

Generalized Brownian Motion is divided between:

- Drift rate → Expected component
- Variance → Random component

$$d\delta_t = \underbrace{\kappa(\theta - \delta_t)dt}_{\text{Expected component}} + \underbrace{\sigma_\delta \sqrt{\delta_t} dB_{\delta,t}}_{\text{Random component}}$$

The Stochastic Model

Axel Buchner - Niklas F. Wagner - Christoph Kaserer

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The model

- The model begins with the specification given in Buchner paper, which is an equation based on a continuous time evolution of the cumulative drawdown:

$$dD_t = \delta_t(C - D_t)$$

- Under the previous condition, the cumulate drawdown can be calculated using the following formula:

$$D_t = C - C \exp\left(-\int_{u=0}^{t} \delta_u du\right)$$

The Stochastic Model

Axel Buchner - Niklas F. Wagner - Christoph Kaserer

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The model

- Therefore, the key of the model is the calculation of the drawdown rate:

$$d\delta_t = \kappa(\theta - \delta_t)dt + \sigma_\delta \sqrt{\delta_t} dB_{\delta,t}$$

- Drawdown rate can be calculated by estimating just 3 parameters:
  - θ → Long Run Mean of the process
  - κ → Reversion Rate
  - σ_δ → Volatility, has to be strictly positive

- The process is a non-negative stochastic process → δ > 0

Estimation Process

I. Discretization of the model & standardization of drawdowns
II. Estimation of the parameters θ & κ
III. Estimation of σ_δ
IV. Calculation of estimated expected cumulative drawdowns
V. Adding the random component
  - Montecarlo Simulation
  - Stress Test as % of Montecarlo realizations

Reference of the estimation:
Buchner, A., Kaserer, C., Wagner N. (2010). 'Private Equity Funds: Valuation, systematic risk and Illiquidity', Working Paper, Version August 2009, pag 39-41

The Stochastic Model

Axel Buchner - Niklas F. Wagner - Christoph Kaserer

Estimation of the expected component

- The discrete representation of the drawdown rate process is:

$$\Delta\delta_t = \kappa(\theta - \delta_t) + \sigma_\delta \sqrt{\delta_t} \times \eta_t$$

  - Where η_t follows a normal distribution N(0,1)

- Firstly, θ and κ are estimated using the concept of Conditional Least Squares (CLS), which involves the minimization of the following formula:

$$\sum_{k=1}^{M} \{\bar{U}_k - \bar{U}_{k-1}(1 - \theta(1 - \exp^{-\kappa}) + \exp^{-\kappa}\, \bar{\delta}_{k-1})\}^2$$

$$\begin{aligned} \bar{d}_k &= \frac{1}{N}\sum_{j=1}^{N} d_{k,j} \\ \bar{U}_{k-1} &= \frac{1}{N}\sum_{j=1}^{N} U_{k-1,j} \\ \bar{\delta}_k &= \frac{\bar{d}_k}{\bar{U}_{k-1}} \end{aligned}$$

Estimation of the random part

- Having the estimations of θ and κ, the parameters γ and η can now be estimated

$$\begin{aligned}\eta_0 &= \frac{\theta}{2\kappa}(1-\exp^{-\kappa})\\ \eta_1 &= \frac{1}{\kappa}(\exp^{-\kappa}-\exp^{-2\kappa})\\ \gamma_0 &= \theta(1-\exp^{-\kappa})\\ \gamma_1 &= \exp^{-\kappa}\end{aligned}$$

- Then, the variance of the drawdown rate per fund is calculated

$$\hat{\sigma}_j^2 = \frac{1}{M}\sum_{k=1}^{M}\frac{[d_{k,j}-(\hat{\gamma}_0+\hat{\gamma}_1\delta_{k-1,j})U_{k-1,j}]^2}{U_{k-1,j}^2(\hat{\eta}_0+\hat{\eta}_1\delta_{k-1,j})}$$

- The estimation of the variance of the drawdown is the calculated by computing the average of the variance of the drawdown rate per fund

$$\hat{\sigma}_\delta^2 = \frac{1}{N}\sum_{j=1}^{N}\hat{\sigma}_j^2$$

The Stochastic Model

Axel Buchner - Niklas F. Wagner - Christoph Kaserer

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From parameters to drawdowns

- Once all the three parameters have been estimated, the cumulated drawdown curve can be calculated as an iterative process, following the next steps:

$$
\begin{aligned}
\Delta\hat{\delta}_1 &= \hat{\kappa}(\hat{\theta} - 0) \\
\hat{\delta}_1 &= = \delta_0 + \Delta\hat{\delta}_1 = 0 + \hat{\kappa} \times \hat{\theta} \\
\hat{D}_1 &= 1 - exp(-\hat{\delta}_1) \\
\hat{U}_1 &= 1 - \hat{D}_1 \\
\Delta\hat{\delta}_2 &= \hat{\kappa}(\hat{\theta} - \hat{\delta}_1) \\
\hat{\delta}_2 &= \max\{\hat{\delta}_1 + \Delta\hat{\delta}_2, 0\} \\
\hat{D}_2 &= 1 - exp(-(\hat{\delta}_1 + \hat{\delta}_2)) \\
\hat{U}_2 &= 1 - \hat{D}_2 \\
\Delta\hat{\delta}_3 &= \hat{\kappa}(\hat{\theta} - \hat{\delta}_2) \\
\hat{\delta}_3 &= \max\{\hat{\delta}_2 + \Delta\hat{\delta}_3, 0\} \\
\hat{D}_3 &= 1 - exp(-(\hat{\delta}_1 + \hat{\delta}_2 + \hat{\delta}_3)) \\
\hat{U}_3 &= 1 - \hat{D}_3
\end{aligned}
$$

And in general:

$$
\begin{aligned}
\Delta\hat{\delta}_k &= \hat{\kappa}(\hat{\theta} - \hat{\delta}_{k-1}) \\
\hat{\delta}_k &= \max\{\hat{\delta}_{k-1} + \Delta\hat{\delta}_k, 0\} \\
\hat{D}_k &= 1 - exp(-\sum_{u=0}^{k} \hat{\delta}_u) \\
\hat{U}_k &= 1 - \hat{D}_k
\end{aligned}
$$

- To perform the simulation with a random path, the random part has to be added to the fixed part

$$\Delta\hat{\delta}_k = \hat{\kappa}(\hat{\theta} - \hat{\delta}_{k-1}) + \hat{\sigma}_\delta \sqrt{\hat{\delta}_{k-1}} \times \epsilon$$

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Estimation results

- The Long Run Mean & Reversion Rate change by cohort
- The standard deviation increases when frequency is low

YEAR	θ	κ	σ_δ
BASELINE	0,7200	0,1800	2,6100
EU	1,4410	0,0742	2,3291
NON-EU	0,3700	0,5200	2,8900
SMALL	0,5620	0,2630	2,6780
LARGE	2,6260	0,0380	2,3680

QUARTER	θ	κ	σ_δ
BASELINE	0,0886	1,1059	7,5151
EU	0,1068	0,2843	9,0998
NON-EU	0,0844	2,1785	8,6453
SMALL	0,0896	1,5743	7,6299
LARGE	0,0928	0,4773	8,3983

MONTHS	θ	κ	σ_δ
BASELINE	0,0313	22,2480	48,6775
EU	0,0278	16,4893	46,1393
NON-EU	0,0384	16,4893	35,6177
SMALL	0,0332	2,9174	17,2036
LARGE	0,0277	2,4824	19,2168

Expected values of the Stochastic Model

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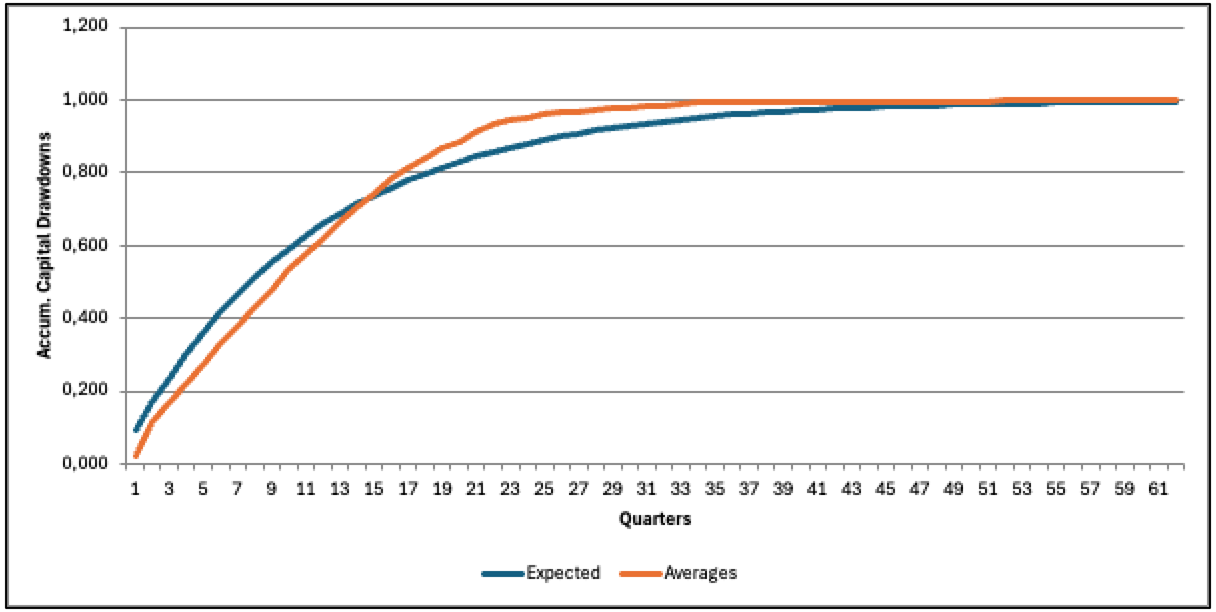
- The averages of the capital drawdown behaves similarly to the expected part of the stochastic model.
- The Stochastic Model increases faster first periods and then stabilises in respect to the averages

$$d\delta_t = \kappa(\theta - \delta_t)dt + \sigma_\delta \sqrt{\delta_t} dB_{\delta,t},$$

ACCUMULATED DRAWDOWNS	EXPECTED PART	AVERAGE VALUES
25%	Q3,2	Q4,5
50%	Q7,7	Q9,4
75%	Q15,6	Q15,1
90%	Q26	Q20,5



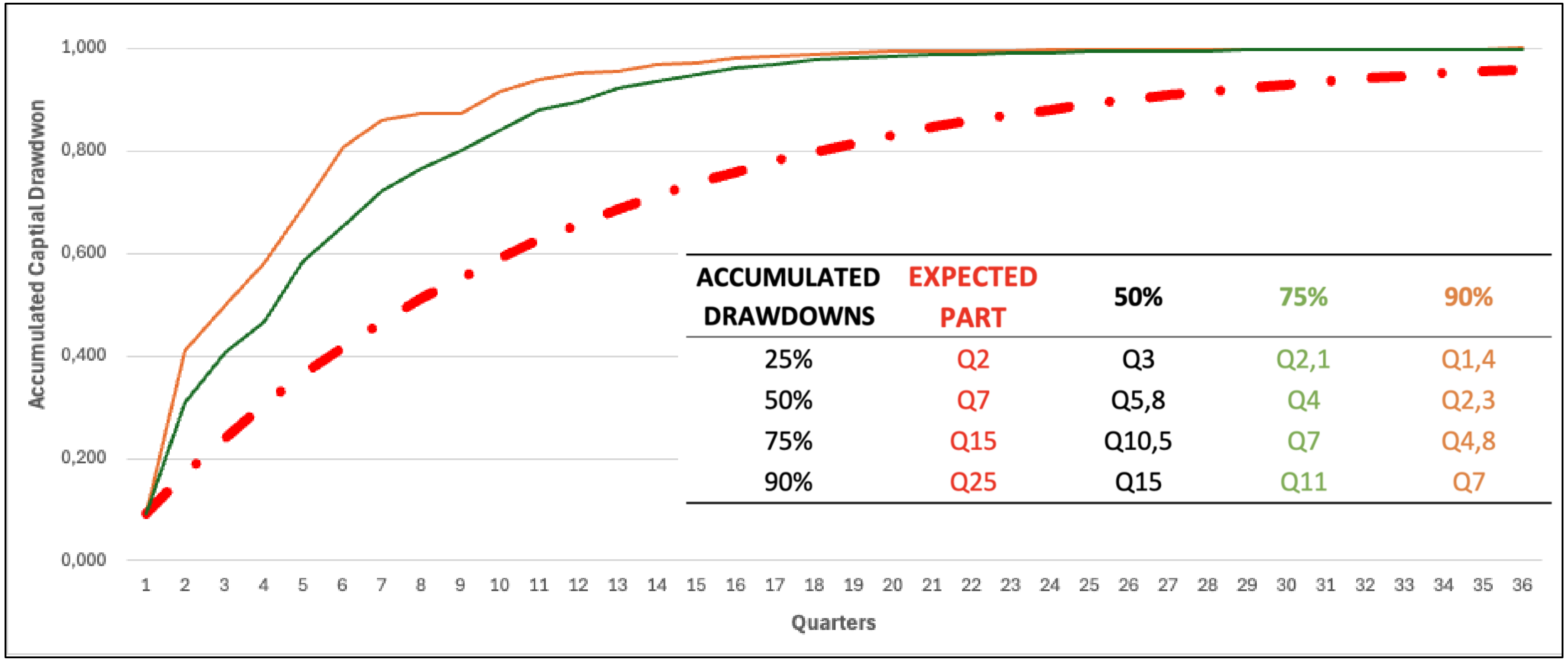
Expected part of the Stoch Model → No randomness for the moment!

Average values (averages)

Stress Test

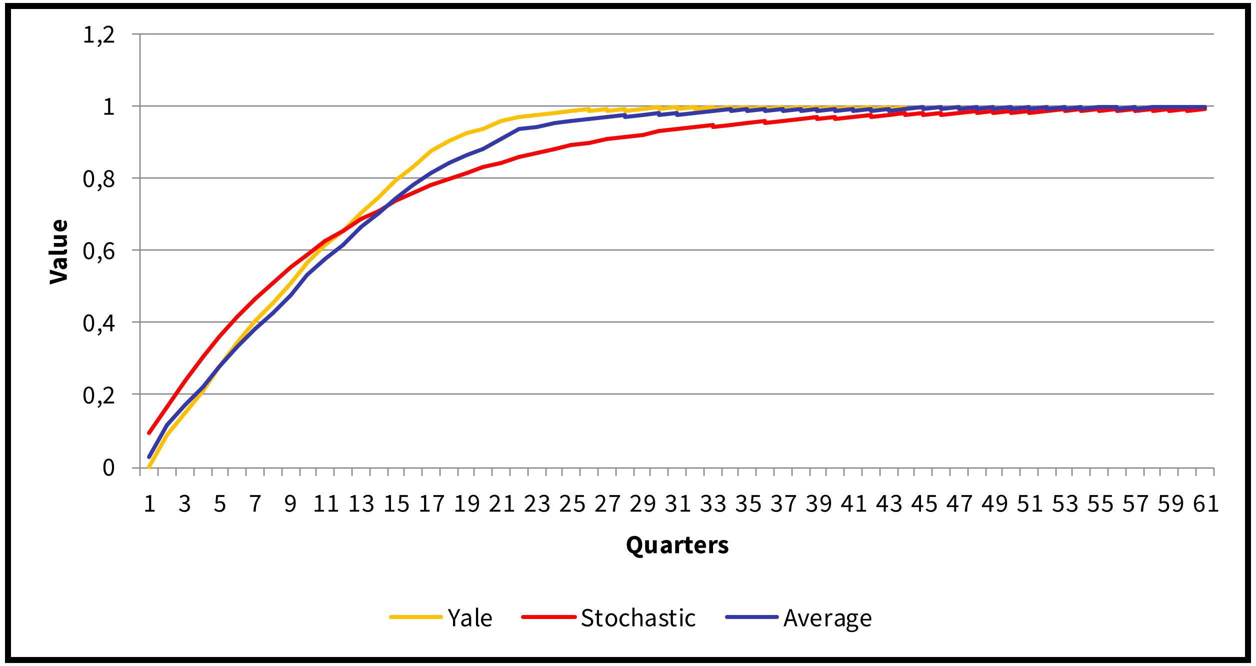
- Based on the Montecarlo Simulation, stress test says that: The probability of an accumulated capital drawdown lower than 0,8 in Q5,1 is 90%.

$$d\delta_t = \kappa(\theta - \delta_t)dt + \sigma_\delta \sqrt{\delta_t} dB_{\delta,t},$$



ACCUMULATED DRAWDOWNS	EXPECTED PART	50%	75%	90%
25%	Q2	Q3	Q2,1	Q1,4
50%	Q7	Q5,8	Q4	Q2,3
75%	Q15	Q10,5	Q7	Q4,8
90%	Q25	Q15	Q11	Q7

Yale vs Stochastic vs Average

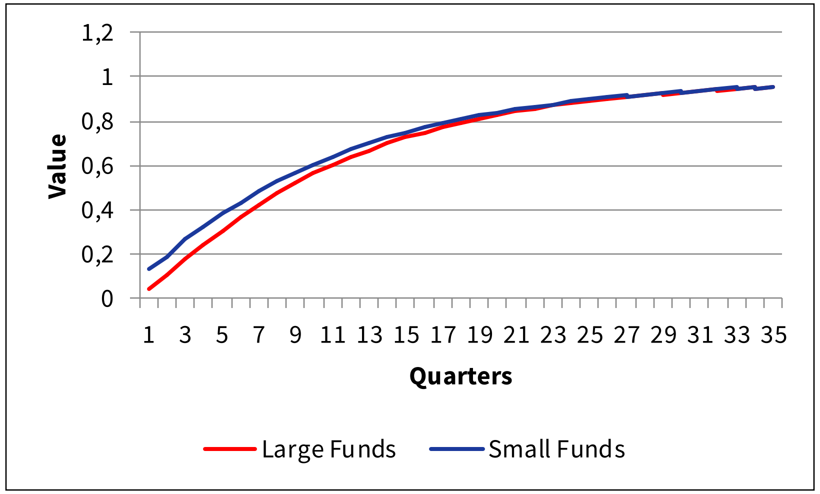


Speed of drawdown			
Cumulated Drawdown	Yale	Stochastic	Average
25%	Q5	Q4	Q5
50%	Q9	Q8	Q10
75%	Q14	Q16	Q16
90%	Q18	Q26	Q21

- The stochastic model introduces a level of uncertainty in the model allowing for more realistic results
- Due to the increase capacity for variability/flexibility in the stochastic model it allows us to highlight the differences between the funds

Cohort Analysis

Expected Values of the Stochastic Model



QUARTER	θ	κ	σ_δ
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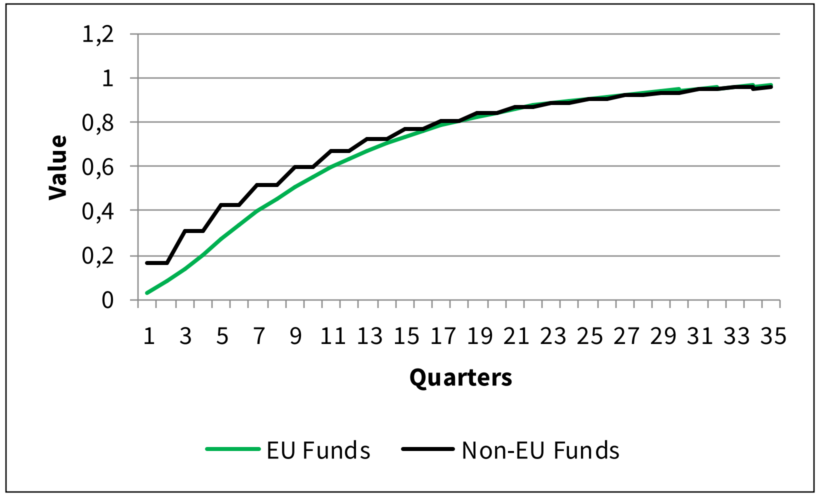

ACCUMULATED DRAWDOWNS	SMALL	LARGE
25%	Q2,8	Q4,1
50%	Q7,6	Q8,6
75%	Q15,1	Q16
90%	Q25,3	Q26

ACCUMULATED DRAWDOWNS	EU	NON-EU
25%	Q4,7	Q2,5
50%	Q9	Q6,8
75%	Q15,5	Q14,5
90%	Q25	Q24,5

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Project | **Dataset** | **Models** | **Results** | **Excel**

Explanation of the Excel

Modelling Cumulative Capital Drawdowns resulting from EIB's investments in Private Equity Funds

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Anex. 1

Impact on cohorts

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Adjustment 1

Removal of funds containing drawdowns with a closed contract type.

107

Removed funds (17.29%)

35% European

65% Non-European

72% Small

28% Large

Adjustment 2

Removal of funds with signature date before 1991 and from 2021 onwards.

83

Removed funds (13.41%)

49% European

51% Non-European

86% Small

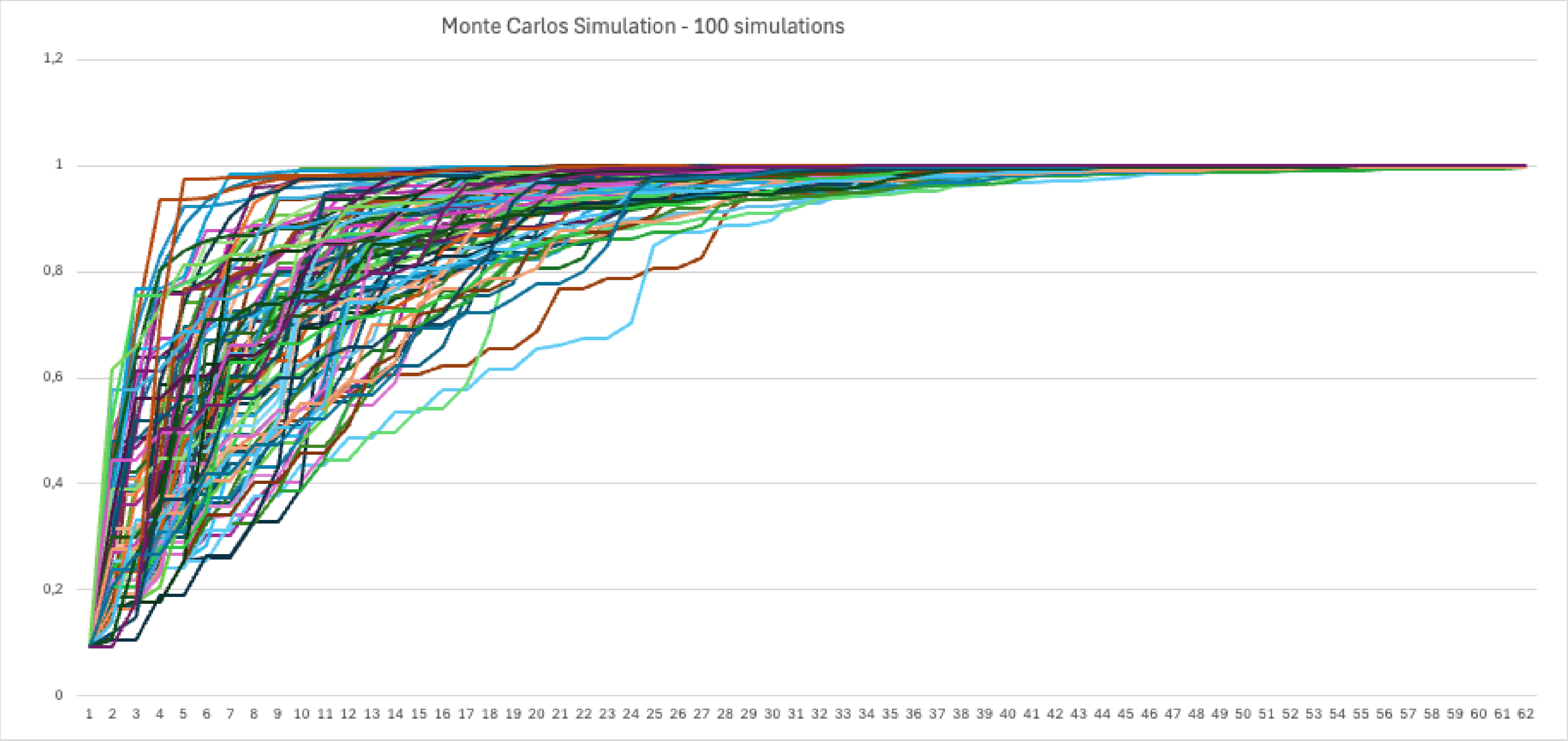
14% Large

Adjustment 3.

Removal of 1 fund with no committed/disbursed capital.

1

Removed fund


Monte Carlos Simulation - 100 simulations
1,2
1
0,8
0,6
0,4
0,2
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62